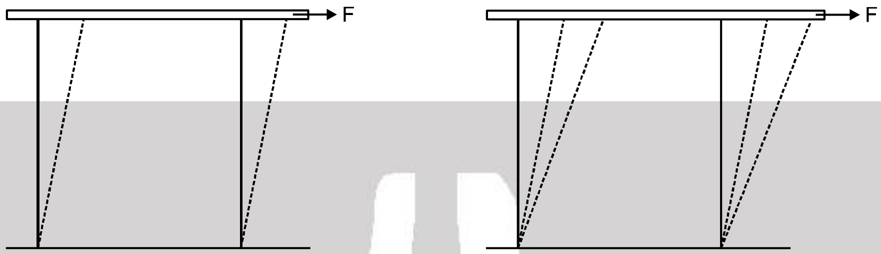
FLUID MECHANICS

Fluid mechanics deals with the behaviour of fluids at rest and in motion. A fluid is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the shear stress may be:
Thus, fluid comprise the liquid and gas (or vapour) phases of the physical forms in which matter exists.
We may alternatively define a fluid as a substance that cannot sustain a shear stress when at rest.



(a) Solid (b) Fluid

1. Density of a Liquid

Density (ρ) of any substance is defined as the mass per unit volume or

$$\rho = \frac{\text{mass}}{\text{volume}} \qquad \rho = \frac{m}{V}$$

2. Relative Density (RD)

In case of a liquid, sometimes an another term relative density (RD) is defined. It is the ratio of density of the substance to the density of water at 4°C. Hence,

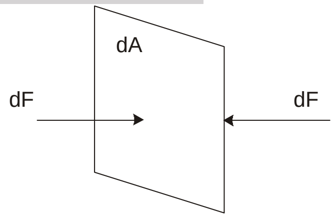
$$RD = \frac{\text{Density of substance}}{\text{Density of water at } 4°C}$$

RD is a pure ratio. So, it has no units. It is also sometimes referred as specific gravity.
Density of water at 4°C in CGS is 1g/cm^3. Therefore, numerically the RD and density of substance (in CGS) are equal. In SI units the density of water at 4°C is 1000 kg/m^3.

Solved Examples

Example 1. Relative density of an oil is 0.8. Find the absolute density of oil in CGS and SI units.
Solution : Density of oil (in CGS) = (RD)g/cm^3 = 0.8 g/cm^3 = 800 kg/m^3

3. Pressure in a Fluid



When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.
While the fluid as a whole is at rest, the molecules that makes up the fluid are in motion, the force exerted by the fluid is due to molecules colliding with their surrounding.
If we think of an imaginary surface within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface, otherwise the surface would accelerate and the fluid would not remain at rest.
Consider a small surface of area dA centered on a point on the fluid, the normal force exerted by the fluid on each side is $dF_{\perp}$. The pressure P is defined at that point as the normal force per unit area, i.e.,

$$P = \frac{dF_{\perp}}{dA}$$

If the pressure is the same at all points of a finite plane surface with area A, then

$$P = \frac{F_{\perp}}{A}$$

where $F_{\perp}$ is the normal force on one side of the surface. The SI unit of pressure is pascal

where 1 pascal = 1Pa = 1.0 N/m^2

One unit used principally in meterology is the Bar which is equal to 10^5 Pa

1 Bar = 10^5 Pa

Note : Fluid pressure acts perpendicular to any surface in the fluid no matter how that surface is oriented. Hence, pressure has no intrinsic direction of its own, its a **scalar**. By contrast, force is a vector with a definite direction.

Atmospheric Pressure (P_0)

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is 1.013×10^5 Pa

Absolute pressure and Gauge Pressure

The excess pressure above atmospheric pressure is usually called gauge pressure and the total pressure is called absolute pressure. Thus,

Gauge pressure = absolute pressure – atmospheric pressure

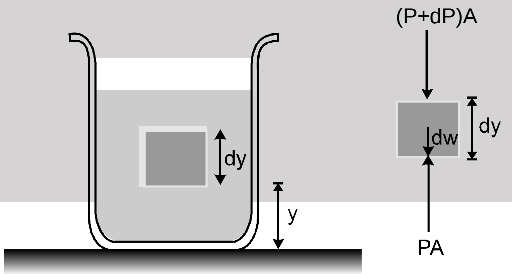
Absolute pressure is always greater than or equal to zero. While gauge pressure can be negative also.

Variation in pressure with depth

If the weight of the fluid can be neglected, the pressure in a fluid is the same throughout its volume. But often the fluid's weight is not negligible and under such condition pressure increases with increasing depth below the surface.

Let us now derive a general relation between the pressure P at any point in a fluid at rest and the elevation y of that point. We will assume that the density ρ and the acceleration due to gravity g are the same throughout the fluid. If the fluid is in equilibrium, every volume element is in equilibrium.

Consider a thin element of fluid with height dy. The bottom and top surfaces each have area A, and they are at elevations y and y + dy above some reference level where y = 0. The weight of the fluid element is



$$dW = (\text{volume})\,(\text{density})\,(g) = (A\,dy)\,(\rho)\,(g)$$

or $$dW = \rho g A dy$$

What are the other forces in y-direction on this fluid element ? Call the pressure at the bottom surface P, the total y component of upward force is PA. The pressure at the top surface is P + dP and the total y-component of downward force on the top surface is (P + dP)A. The fluid element is in equilibrium, so the total y component of force including the weight and the forces at the bottom and top surfaces must be zero.

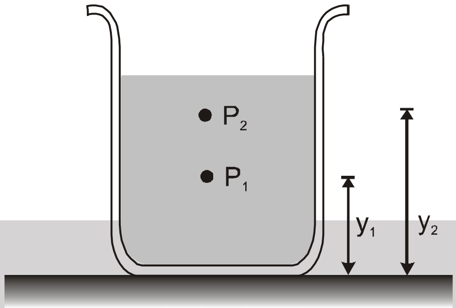
$$\Sigma F_y = 0$$

$$\therefore \quad PA - (P + dP)A - \rho gA\,dy = 0 \quad \text{or} \quad \frac{dP}{dy} = -\rho g$$

This equation shows that when y increases, P decreases, i.e., as we move upward in the fluid pressure decreases.

If P_1 and P_2 be the pressures at elevations y_1 and y_2 and if ρ and g are constant, then integration Equation (i), we get

or $\quad P_2 - P_1 = -\rho g (y_2 - y_1)$ ....(ii)



It's often convenient to express Equation (ii) in terms of the depth below the surface of a fluid. Take point 1 at depth h below the surface of fluid and let P represents pressure at this point. Take point 2 at the surface of the fluid, where the pressure is P_0 (subscript for zero depth). The depth of point 1 below the surface is,

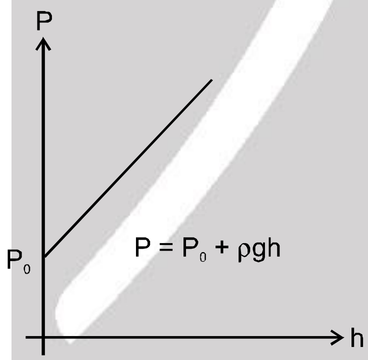
$h = y_2 - y_1$

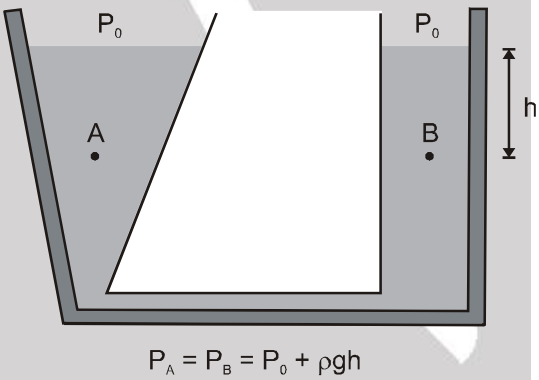
and equation (ii) becomes

$P_0 - P = -\rho g (y_2 - y_1) = -\rho gh$

$\therefore \quad P = P_0 + \rho gh$ ....(iii)

Thus pressure increases linearly with depth, if ρ and g are uniform. A graph between P and h is shown below.

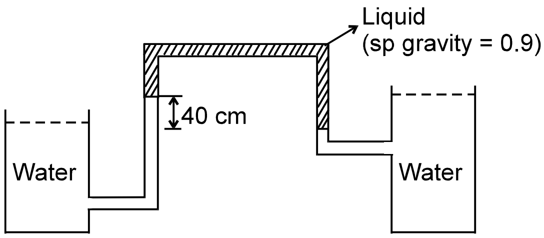




Further, the pressure is the same at any two points at the same level in the fluid. The shape of the container does not matter.

Solved Examples

Example 2. The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated.



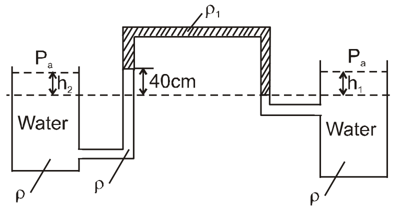
Solution : $P_a + h_1 \rho g - 40\rho_1 g + 40\rho g = P_a + h_2 \rho g$

$h_2 \rho g - h_1 \rho g = 40 \rho g - 40 \rho_1 g$

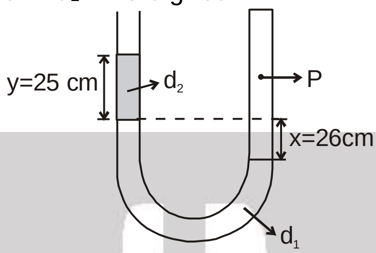
as $\rho_1 = 0.9\rho$

$(h_2 - h_1) \rho g = 40\rho g - 36\rho g$

$h_2 - h_1 = 4$ cm



Example 3. In a given U-tube (open at one-end) find out relation between P and P_a.
Given $d_2 = 2 \times 13.6$ gm/cm^3 $d_1 = 13.6$ gm/cm^3



Solution : Pressure in a liquid at same level is same i.e. at A – A–,

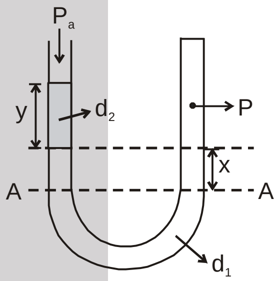
$P_a + d_2 y g + x d_1 g = P$

In C.G.S.

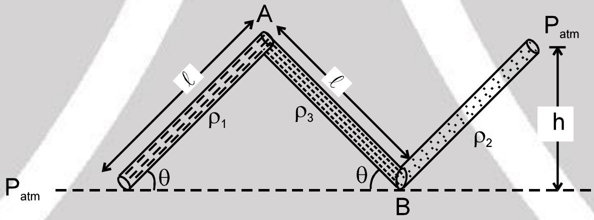
$P_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = P$

$P_a + 13.6 \times g\,[50 + 26] = P$

$2P_a = P$ $\qquad [P_a = 13.6 \times g \times 76]$



Example 4. Find out pressure at points A and B. Also find angle 'θ'.

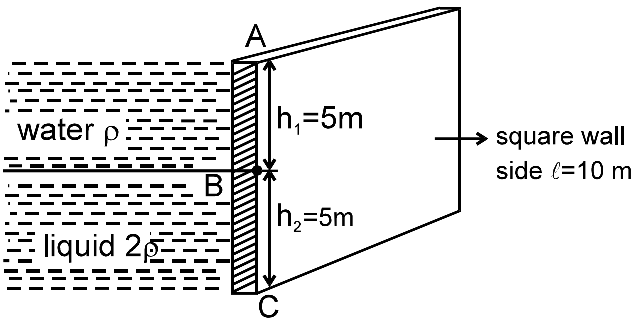


Solution :
Pressure at A – $\qquad P_A = P_{atm} - \rho_1 gl \sin\theta$
Pressure at B $\qquad P_B = P_{atm} + \rho_2 gh\ \theta$
But P_B is also equal to $\qquad P_B = P_A + \rho_3 gl \sin\theta$
Hence - $\quad P_{atm} + \rho_2 gh = P_A + \rho_3 gl \sin\theta$

$P_{atm} + \rho_2 gh = P_{atm} - \rho_1 gl \sin\theta + \rho_3 gl \sin\theta$

$\sin\theta = \dfrac{\rho_2 h}{(\rho_3 - \rho_1)\ell}$.

Example 5. Water and liquid is filled up behind a square wall of side ℓ. Find out



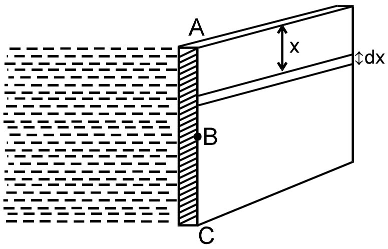
(a) Pressures at A, B and C $\qquad$ (b) Forces in part AB and BC

Solution : (a) As there is no liquid above 'A',

So pressure at A, $P_A = 0$

Pressure at B, $P_B = \rho g h_1$

Pressure at C, $P_C = \rho g h_1 + 2\rho g h_2$



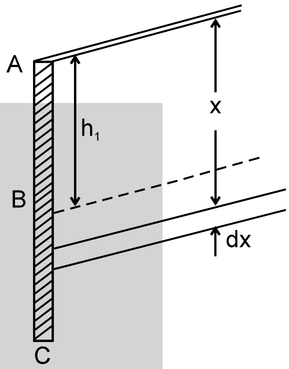
(b) Force at A = 0 Take a strip of width 'dx' at a depth 'x' in part AB.

Pressure is equal to $\rho g x$.

Force on strip = pressure × area

$dF = \rho g x \, \ell dx$

Total force upto B



$$F = \int_0^{h_1} \rho g x \ell dx = \frac{\rho g x \ell h_1^2}{2} = \frac{1000 \times 10 \times 10 \times 5 \times 5}{2}$$

$= 1.25 \times 10^6$ N

In part BC for force take a elementary strip of width dx in portion BC. Pressure is equal to

$= \rho g h_1 + 2\rho g(x - h_1)$

Force on elementary strip = pressure × area

$dF = [\rho g h_1 + 2\rho g(x - h_1)] \, \ell \, dx$

Total force on part BC

$$F = \int_{h_1}^{\ell} [\rho g h_1 + 2\rho g(x - h_1)] \ell dx = \left[\rho g h_1 x + 2\rho g \left[\frac{x^2}{2} - h_1 x\right]\right]_{h_1}^{\ell} \ell$$

$$= \rho g h_1 h_2 \, \ell + 2\rho g \ell \left[\frac{\ell^2 - h_1^2}{2} - h_1 \ell + h_1^2\right]$$

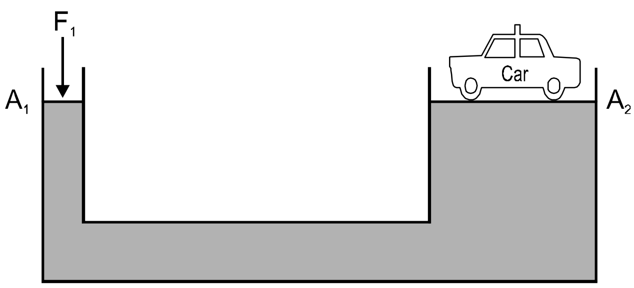
$$= \rho g h_1 h_2 \ell + \frac{2\rho g \ell}{2} [\ell^2 + h_1^2 - 2h_1 \ell] = \rho g h_1 h_2 \ell + \rho g \ell (\ell - h_1)^2$$

$$= \rho g h_2 \ell [h_1 + h_2] = \rho g h_2 \ell^2 = 1000 \times 10 \times 5 \times 10 \times 10 = 5 \times 10^6 \text{ N}$$

Pascal's Law

It states that "pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel".

A well known application of Pascal's law is the hydraulic lift used to support or lift heavy objects. It is schematically illustrated in figure.



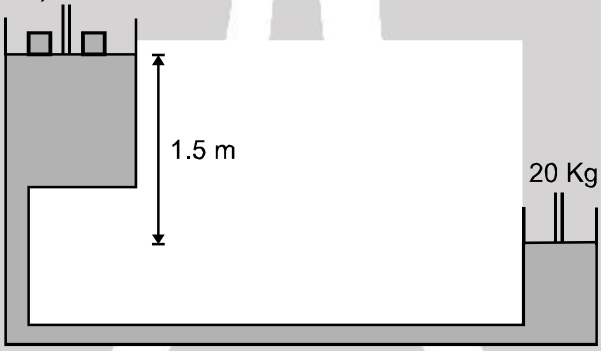
A piston with small cross section area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $P = \frac{F_1}{A_1}$ is transmitted through the connection pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad F_2 = \frac{A_2}{A_1}.F_1$$

Now, since $A_2 > A_1$, therefore, $F_2 > F_1$. Thus hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, elevators and hydraulic brakes all are based on this principle.

Solved Examples

Example 6. Figure shows a hydraulic press with the larger piston of diameter 35 cm at a height of 1.5 m relative to the smaller piston of diameter 10 cm. The mass on the smaller piston is 20 kg. What is the force exerted on the load by the larger piston ? The density of oil in the press is 750 kg/ m^3. (Take g = 9.8m/s^2)



Solution : Pressure on the smaller piston $= \frac{20 \times 9.8}{\pi \times (5 \times 10^{-2})^2} N/m^2$

Pressure on the larger piston $= \frac{F}{\pi \times (17.5 \times 10^{-2})^2} N/m^2$

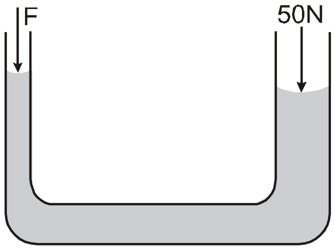
The difference between the two pressures = $h\rho g$

where h = 1.5 m and ρ = 750 kg/m^3

Thus, $\frac{20 \times 9.8}{\pi \times (5 \times 10^{-2})^2} - \frac{F}{\pi \times (17.5 \times 10^{-2})^2} = 1.5 \times 750 \times 9.8 = 11025 \Rightarrow F = 1.3 \times 10^3$ N

Note : atmospheric pressure is common to both pistons and has been ignored.

Example 7. The area of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2 respectively (figure). A force of 50 N is applied on the water in the thicker arm. What force should be applied on the water in the thinner arm so that the water may remain in equilibrium?

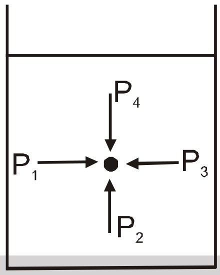


Solution : In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If the atmospheric pressure is P and a force F is applied to maintain the equilibrium, the pressures are $P_0 + \frac{50N}{10cm^2}$ and $P_0 + \frac{F}{1\ cm^2}$ respectively.

This givens F = 5 N.

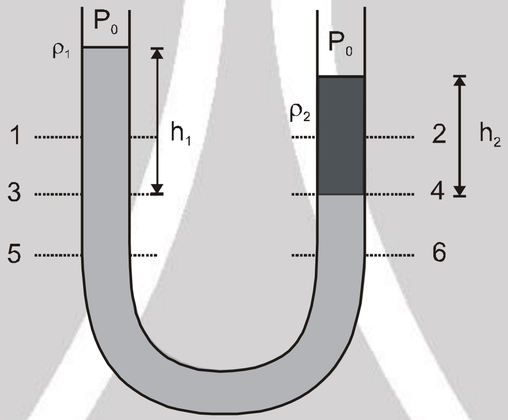
Important points in Pressure

1. At same point on a fluid pressure is same in all direction. In the figure,
$P_1 = P_2 = P_3 = P_4$



2. Forces acting on a fluid in equilibrium have to be perpendicular to its surface. Because it cannot sustain the shear stress.

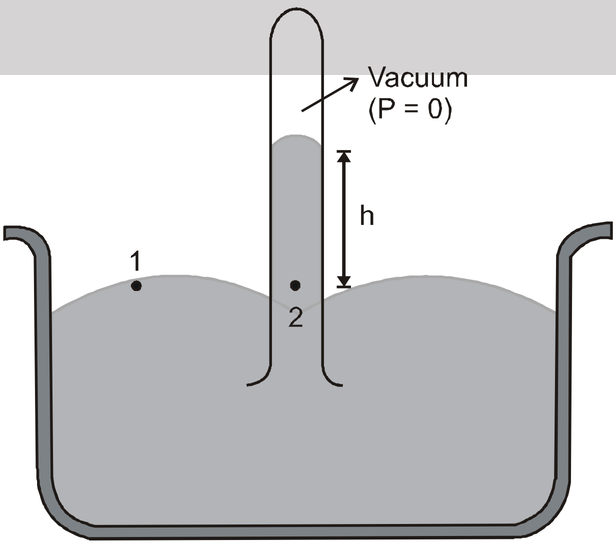
3. In the **same liquid** pressure will be same at all points at the same level. For example, in the figure:



$P_1 \neq P_2$
$P_3 = P_4$ and $P_5 = P_6$
Further $P_3 = P_4$

$\therefore \quad P_0 + \rho_1 gh_1 = P_0 + \rho_2 gh_2 \quad$ or $\quad \rho_1 h_1 = \rho_2 h_2 \quad$ or $\quad h \propto \frac{1}{\rho}$

4. **Torricelli Experiment (Barometer) :**
It is a device used to measure atmospheric pressure .In principle any liquid can be used to fill the barometer, but mercury is the substance of choice because its great density makes possible an instrument of reasonable size.



$P_1 = P_2$

Here, P_1 = atmospheric pressure (P_0)

and $P_2 = 0 + \rho gh = \rho gh$

$P_0 = \rho gh$

Here ρ = density of mercury

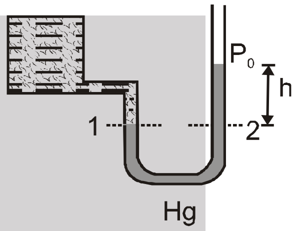
Thus, the mercury barometer reads the atmosheric pressure (P_0) directly from the height of the mercury column.

For example if the height of mercury in a barometer is 760 mm. then atmospheric pressure will be ,

$P_0 = \rho gh = (13.6 \times 10^3)(9.8)(0.760) = 1.01\times10^5$ N/m^2

5. Manometer :

It is a device used to measure the pressure of a gas inside a container. The U- shaped tube often contains mercury



$P_1 = P_2$

Here P_1 = pressure of the gas in the container (P)

and P_2 = atmospheric pressure (P_0) + ρgh

$P = P_0 + h\rho g$

This can also be written as

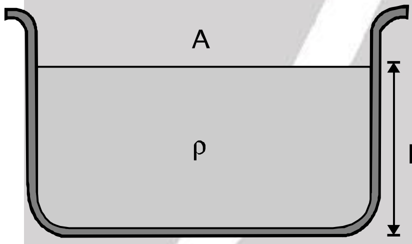
$P - P_0$ = gauge pressure = ρgh

Here, ρ is the density of the liquid used in U - tube.

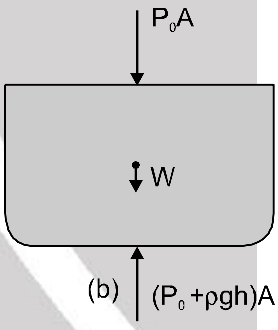
Thus by measuring h we can find absolute (or gauge) pressure in the vessel.

6. Free body diagram of a liquid :

The free body diagram of the liquid(showing the vertical forces only) is shown in fig (b) For the equilibrium of liquid .



(a)



(b)

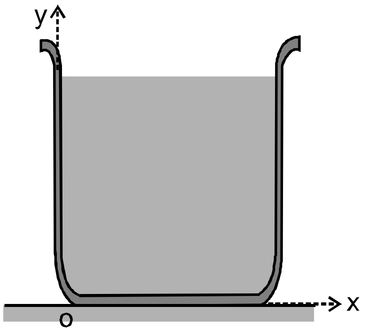
Net downward force = net upward force

$\therefore$ $P_0A + W = (P_0 + \rho gh)A$ or $W = \rho ghA$

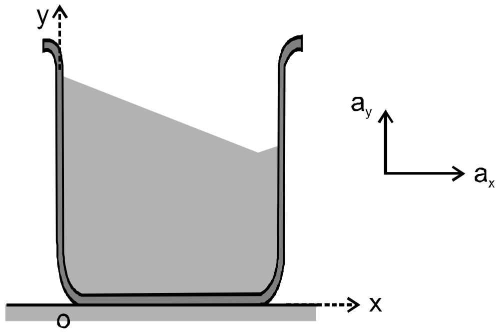
7. Pressure Difference in Accelerating Fluids

Consider a liquid kept at rest in a beaker as shown in figure (a). In this case we know that pressure do not change in horizontal direction (x-direction), it decreases upwards along y-direction So, we can write the equations,

$$\frac{dP}{dx} = 0 \text{ and } \frac{dP}{dy} = \rho g$$



(a)



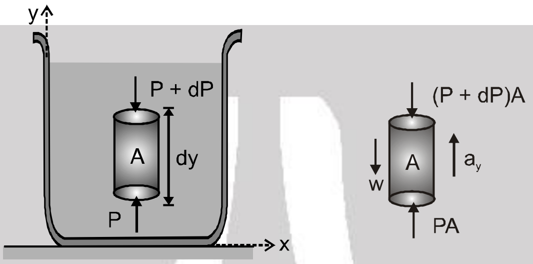
(b)

But, suppose the beaker is accelerated and it has components of acceleration a_x and a_y in x and y-directions respectively, then the pressure decreases along both x and y directions. The above equation in that case reduces to

$$\frac{dP}{dx} = -\rho a_x \text{ and } \frac{dP}{dy} = -\rho(g + a_y)$$

These equations can be derived as under. Consider a beaker filled with some liquid of density ρ accelerating upwards with an acceleration a_y along positive y-direction, Let us draw the free body diagram of a small element of fluid of area A and length dy as shown in figure . Equation of motion for this element is,

$PA - W - (P + dP)\,A = (\text{mass})(a_y)$ or $-W - (dP)\,A = (A\rho\, dy)(a_y)$

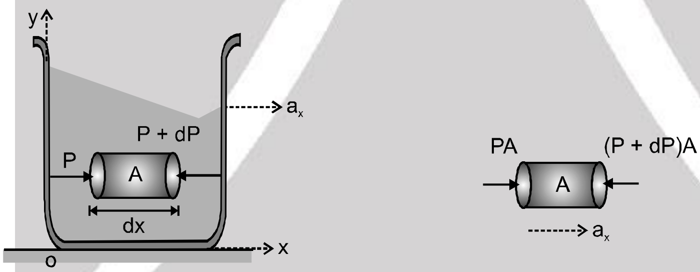


(a)

or $-(A\rho g\, dy) - (dP)\,A = (A\rho\, dy)(a_y)$ or $\frac{dP}{dy} = -\rho(g + a_y)$

Similarly, if the beaker moves along positive x-direction with acceleration a_x, the equation of motion for the fluid element shown in figure is

$PA - (P + dP)\,A = (\text{mass})\,(a_x)$ or $-(dP)\,A = (A\rho\, dx)\,a_x$ or $\frac{dP}{dx} = -\rho a_x$

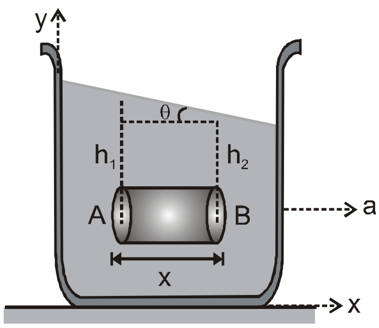


(a)

8. Free Surface of a Liquid Accelerated in Horizontal Direction

Consider a liquid placed in a beaker which is accelerating horizontally with an acceleration 'a'. Let A and B be two points in the liquid at a separation x in the same horizontal line. As we have seen in this case

$dp = \rho a\, dx$ or $\frac{dP}{dx} = -\rho a$



Integrating this with proper limits, we get $P_A - P_B = \rho ax$

Furter $P_A = P_0 + \rho g h_1$

and $P_B = P_0 + \rho g h_2$

substituting in Eq. (iii) we get

$\rho g\,(h_1 - h_2) = \rho ax$

$$\frac{h_1 - h_2}{x} = \frac{a}{g} = \tan\theta$$

$$\tan\theta = \frac{a}{g}$$

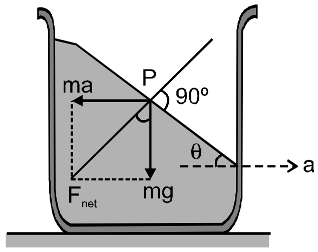
Alternate Method

Consider a fluid particle of mass m at point P on the surface of liquid. From the accelerating frame of reference, two forces are acting on it,

(i) pseudo force(ma) (ii) Weight (mg)

As we said earlier also, net force in equilibrium should be perpendicular to the surface.

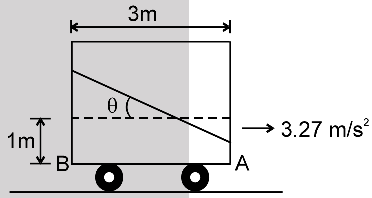
$\therefore \tan\theta = \frac{ma}{mg}$ or $\tan\theta = \frac{a}{g}$



Solved Examples

Example 8. An open rectangular tank 1.5 m wide 2m deep and 2m long is half filled with water. It is accelerated horizontally at 3.27 m/sec^2 in the direction of its length. Determine the depth of water at each end of tank. [g = 9.81 m/sec^2]

Solution : $\tan\theta = \frac{a}{g} = \frac{1}{3}$



Depth at corner 'A'

= 1 – 1.5 tanθ = 0.5 m **Ans.**

Depth at corner 'B'

= 1 + 1.5 tan θ = 1.5 m **Ans.**

9. Archimedes 'Principle

If a heavy object is immersed in water, it seems to weigh less than when it is in air. This is because the water exerts an upward force called buoyant force. It is equal to the weight of the fluid displaced by the body **A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.**

This result known as **Archimedes 'principle.**

Thus, the magnitude of buoyant force (F) is given by, $F = V_i \rho_L g$

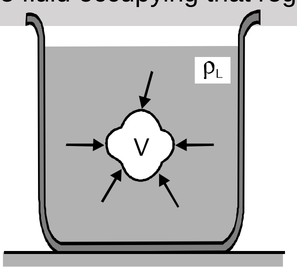
Here, V_i = immersed volume of solid

ρ_L = density of liquid

and g = acceleration due to gravity

Proof

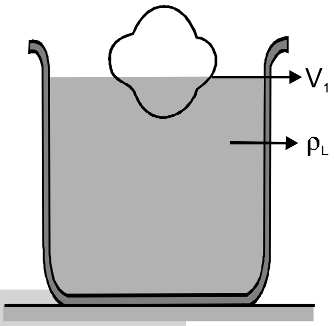
Consider an arbitrarily shaped body of volume V placed in a container filled with a fluid of density ρ_L. The body is shown completely immersed, but complete immersion is not essential to the proof. To begin with, imagine the situation before the body was immersed. The region now occupied by the body was filled with fluid, whose weight was $V\rho_L g$. Because the fluid as a whole was in hydrostatic equilibrium, the net upwards force (due to difference in pressure at different depths) on the fluid in region was equal to the weight of the fluid occupying that region.



Now, consider what happens when the body has displaced the fluid. The pressure at every point on the surface of the body is unchanged from the value at the same location when the body was every point on. This is because the pressure at any point depends only on the depth of that point the surface. Hence, the net force exerted by the surrounding fluid on the body is exactly the same as that exerted on the region before the body was present. But we now latter to be $V\rho_L g$, the weight of the displaced fluid Hence, this must also be the buoyant force exerted of the body. Archimedes' principle is thus proved.

10. Law of Floatation

Consider an object of volume V and density ρ_s floating in a liquid of density ρ_L. Let V_i be the object immersed in the liquid.



For equilibrium of object,

Weight = Upthrust

$\therefore \quad V\rho_s g = V_i \rho_L g$

$\therefore \quad \frac{V_i}{V} = \frac{\rho_s}{\rho_L}$

This is the fraction of volume immersed in liquid.

Percentage of volume immersed in liquid = $\frac{V_i}{V} \times 100 = \frac{\rho_s}{\rho_L} \times 100$

Three possibilities may now arise.

(i) If $\rho_s < \rho_L$, only fraction of body will be immersed in the liquid. This fraction will be given by the above equation.

(ii) If $\rho_s = \rho_L$, the whole of the rigid body will be immersed in the liquid. Hence the body remains floating in the liquid wherever it is left.

(iii) If $\rho_s > \rho_L$, the body will sink.

Apparent Weight of a Body inside a Liquid

If a body is completely immersed in a liquid its effective weight gets decreased. The decrease in its weight is equal to the upthrust on the body. Hence,

$W_{app} = W_{actual}$ – Upthrust

or $\quad W_{app} = V\rho_s g - V\rho_L g$

Here, V = total volume of the body

ρ_s = density of body

and ρ_L = density of liquid

Thus, $W_{app} = Vg(\rho_s - \rho_L)$

If the liquid in which body is immersed, is water, then

$$\frac{\text{Weight in air}}{\text{Decrease in weight}} = \text{Relative density of body (R.D)}$$

This can be shown as under :

$$\frac{\text{Weight in air}}{\text{Decrease in weight}} = \frac{\text{Weight in air}}{\text{Upthrust}} = \frac{V\rho_s g}{V\rho_w g} = \frac{\rho_s}{\rho_w} = \text{RD}$$

Buoyant Force in Accelerating Fluids

Suppose a body is dipped inside a liquid of density ρ_L placed in an elevator moving with an acceleration $\vec{a}$.The buoyant force F in this case becomes,

$F = V\rho_L g_{eff}$

Here , $g_{eff} = |\vec{g} - \vec{a}|$

For example, if the lift is moving upwards with an acceleration a, value of g_{eff} is g + a and if it is moving downwards with acceleration a, the g_{eff} is g – a In a freely falling lift g_{eff} is zero (as a = g) and hence, net buoyant force is zero. This is why, in a freely falled with some liquid, the air bubbles do not rise up (which otherwise move up due to buoyant force).

Solved Examples

Example 9. Density of ice is $900 kg/m^3$. A piece of ice is floating in water of density $1000\ kg/m^3$. Find the fraction of volume of the piece of ice out side the water.

Solution : Let V be the total volume and V_i the volume of ice piece immersed in water. For equilibrium of ice piece,

weight = upthrust

$\therefore \quad V\rho_i g = V_i \rho_w g$

Here ρ_i = density of ice = $900 kg/m^3$

and ρ_w = density of water = $1000 kg/m^3$

Substituting in above equation,

$$\frac{V_i}{V} = \frac{900}{1000} = 0.9$$

i.e, the fraction of volume outside the water,

f = 1– 0.9 = 0.1

Example 10. A piece of ice is floating in a glass vessel filled with water. How the level of water in the vessel change when the ice melts ?

Solution : Let m be the mass of ice piece floating in water.

In equilibrium, weight of ice piece = upthrust

$mg = V_i \rho_w g$

or $\quad V_i = \frac{m}{\rho_w}$

Here , V_i is the volume of ice piece immersed in water

When the ice melt, let V be the volume of water formed by m mass of ice. Then,

$$V_i = \frac{m}{\rho_w}$$

From Eqs. (i) and (ii) we see that

$V_i = V$

Hence, the level will not change.

Example 11. A piece of ice having a stone frozen in it floats in a glass vessel filled with water. How will the level of water in the vessel change when the ice melts ?

Solution : Let, m_1 = mass of ice ,

m_2 = mass of stone

ρ_s = density of stone

and ρ_w = density of water

In equilibrium, when the piece of ice floats in water , weight of (ice + stone) = upthrust

$(m_1 + m_2)g = V_i \rho_w g \qquad \therefore V_i = \frac{m_1}{\rho_w} + \frac{m_2}{\rho_w}$

Here, V_i = Volume of ice immersed

when the ice melts m_1 mass of ice converts into water and stone of mass m_2 is completely submerged .

Volume of water formed by m_1 mass of ice,

$$V_1 = \frac{m_1}{\rho_w}$$

Volume of stone (which is also equal to the volume of water displaced)

$$V_2 = \frac{m_2}{\rho_s}$$

Since, $\rho_s > \rho_w$ Therefore, $V_1 + V_2 < V_i$

or, the level of water will decrease .

Example 12. An ornament weighing 50 g in air weighs only 46 g is water. Assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it. Specific gravity of gold is 20 and that of copper is 10.

Solution : Let m be the mass of the copper in ornament . Then mass of gold in it is (50 – m).

Volume of copper $V_1 = \frac{m}{10}$ $\left(\text{volume} = \frac{\text{mass}}{\text{density}}\right)$

and volume of gold $V_2 = \frac{50-m}{20}$

when immersed in water (ρ_w = 1g/cm^3)

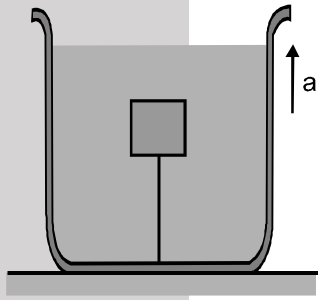
Decrease in weight = upthrust

$\therefore$ (50 – 46)g = $(V_1 + V_2)\rho_w g$

or $4 = \frac{m}{10} + \frac{50-m}{20}$ or 80 = 2m + 50 – m

$\therefore$ m = 30g

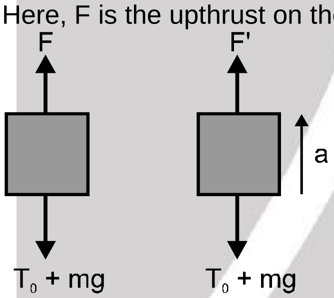
Example 13. The tension in a string holding a solid block below the surface of a liquid (of density greater then that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension in the string if the system has an upward acceleration a ?



Solution : Let m be the mass of block. Initially for the equilibrium of block,

F = T_0 + mg

Here, F is the upthrust on the block .



Where the lift is accelerated upwards, g_{eff} becomes g + a instead of g.

Hence, $F' = F\left(\frac{g+a}{g}\right)$

From Newton's second law, F' – T – mg = ma

Solving Eqs. (i) , (ii) and (iii),we get

$T = T_0\left(1 + \frac{a}{g}\right)$

Example 14. A metal piece of mass 10 g is suspended by a vertical spring. The spring elongates 10 cm over its natural length to keep the piece in equilibrium. A beaker containing water is now placed below the piece so as to immerse the piece completely in water. Find the elongation of the spring. Density of metal = 9000 kg/m^3. Take g = 10 m/s^2.

Solution : Let the spring constant be k. When the piece is hanging in air, the equilibrium condition gives

k (10 cm) = (0.01 kg) (10 m/s^2)

or k (10 cm) = 0.1 N. .....(i)

The volume of the metal piece

$= \frac{0.01\text{kg}}{9000\,\text{kg/m}^3} = \frac{1}{9} \times 10^{-5}$ m^3.

This is also the volume of water displaced when the piece is immersed in water. The force of buoyancy

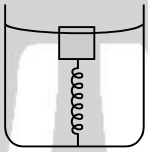
= weight of the liquid displaced = $\frac{1}{9} \times 10^{-5}$ m^3 × (1000 kg/m^3) × (10 m/s^2) = 0.011 N.

If the elongation of the spring is x when the piece is immersed in water, the equilibrium condition of the piece gives,

kx = 0.1 N – 0.011 N = 0.089 N. .....(ii)

By (i) and (ii), $x = \frac{0.089}{10}$ cm = 0.0089 cm.

Example 15. A cubical block of plastic of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of plastic = 800 kg/m^3 and spring constant of the spring = 100 N/m. Take g = 10 m/s^2.



Solution : The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside water = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water

= volume of the block = 27 × 10^{-6} m^3.

Hence, the force of buoyancy

= (27 × 10^{-6} m^3) × 1(1000 kg/m3) × (10 m/s^2) = 0.27 N.

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

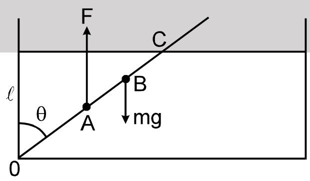
= 100 N/m × 0.6 cm = 0.6 N.

The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

W′ = (27 × 10^{-6} m) × (800 kg/m^3) × (10 m/s^2) = 0.22 N.

Thus, W = 0.27 N + 0.6 N – 0.22 N = 0.65 N.

Example 16. A wooden plank of length 2ℓ m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water up to a height of ℓ m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0)



Solution : The forces acting on the plank are shown in the figure. The height of water level is ℓ. The length of the plank is 2ℓ. The weight of the plank acts through the centre B of the plank. We have OB = ℓ. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have OA = $\frac{OC}{2} = \frac{\ell}{2\cos\theta}$.

Let the mass per unit length of the plank be ρ.

Its weight mg = 2$\ell\rho$g.

The mass of the part OC of the plank = $\left(\frac{\ell}{\cos\theta}\right)\rho$.

The mass of water displaced = $\frac{1}{0.5}\,\frac{\ell}{\cos\theta}\,\rho = \frac{2\ell\rho}{\cos\theta}$.

The buoyant force F is, therefore, F = $\frac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, mg (OB) sinθ = F(OA) sinθ

or, $(2\ell\rho)\ell = \left(\frac{2\ell\rho}{\cos\theta}\right)\left(\frac{\ell}{2\cos\theta}\right)$ or, $\cos^2\theta = \frac{1}{2}$ or, $\cos\theta = \frac{1}{\sqrt{2}}$, or, $\theta = 45°$.

Example 17. A cylindrical block of wood of mass m, radius r & density ρ is floating in water with its axis vertical. It is depressed a little and then released. If the motion of the block is simple harmonic. Find its frequency.

Solution : Suppose a height h of the block is dipped in the water in equilibrium position. If r be the radius of the cylindrical block, the volume of the water displaced = $\pi r^2 h$. For floating in equilibrium,

$\pi r^2 h\rho g = W$ ....(i)

where ρ is the density of water and W the weight of the block.

Now suppose during the vertical motion, the block is further dipped through a distance x at some instant. The volume of the displaced water is $\pi r^2 (h + x)$. The forces acting on the block are, the weight W vertically downward and the buoyancy $\pi r^2(h + x)\rho g$ vertically upward.

Net force on the block at displacement x from the equilibrium position is

$F = W - \pi r^2 (h + x)\rho g = W - \pi r^2 h\rho g - \pi r^2\rho x g$

Using (i) $F = -\pi r^2 \rho g x = -kx$,

where $k = \pi r^2 \rho g$.

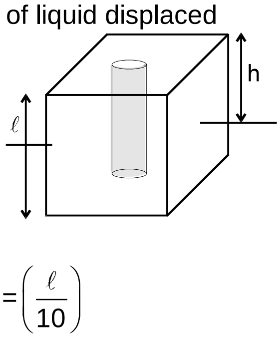
Thus, the block executes SHM with frequency.

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\pi r^2\rho g}{m}}.$$

Example 18. A large block of ice cuboid of height 'ℓ' and density $\rho_{ice} = 0.9\,\rho_w$, has a large vertical hole along its axis. This block is floating in a lake. Find out the length of the rope required to raise a bucket of water through the hole.

Solution : Let area of ice-cuboid excluding hole = A

weight of ice block = weight of liquid displaced



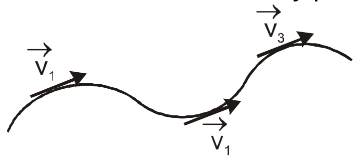
$A\,\rho_{ice}\,\ell\,g = A\,\rho_w\,(\ell - h)\,g$

$\frac{9\ell}{10} = \ell - h \Rightarrow h = \ell - \frac{9\ell}{10} = \left(\frac{\ell}{10}\right)$

11. Flow of Fluids

Steady Flow

If the velocity of fluid particles at any point does not vary with time, the flow is said to be steady. Steady flow is also called streamlined or laminar flow .The velocity points may be different. Hence in the figure,

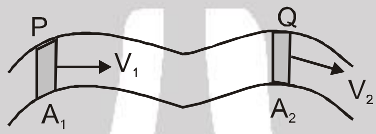


$\vec{v}_1$ = constant, $\vec{v}_2$ = constant, $\vec{v}_3$ = constant

but $\vec{v}_1 \neq \vec{v}_2 \neq \vec{v}_3$

12. Principle of Continuity

It states that, when an incompressible and non-viscous liquid flows in a stream lined motion through a tube of non- uniform cross section ,then the product of the area of cross section and the velocity of flow is same at every point in the tube .



Thus, $A_1v_1 = A_2\,v_2$

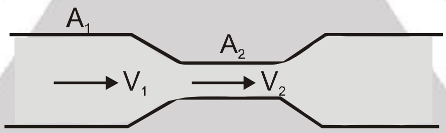
or Av = constant or $v \propto \frac{1}{A}$

This is basically the law of conservation of mass in fluid dynamics.

Proof

Let us consider two cross sections P and Q of area A_1 and A_2 of a tube through which a fluid is flowing. Let v_1 and v_2 be the speeds at these two cross sections. Then being an incompressible fluid, mass of fluid going through P in a time interval Δt = mass of fluid passing through Q in the same interval of time Δt

$\therefore$ $A_1v_1\rho\,\Delta t = A_2v_2\rho\,\Delta t$ or $A_1\,v_i = A_2v_2$



Therefore, the velocity of the liquid is smaller in the wider part of the tube and larger in the narrower parts.

or $v_2 > v_1$ as $A_2 < A_1$

Note : The product Av is the volume flow rate $\frac{dV}{dt}$, the rate at which volume crosses a section of the tube. Hence $\frac{dV}{dt}$ = volume flow rate = Av

The mass flow rate is the mass flow per unit time through a cross section. This is equal to density (ρ) times the volume flow rate $\frac{dV}{dt}$.

we can generalize the continuity equation for the case in which the fluid is not incompressible . If ρ_1 and ρ_2 are the densities at sections1 and 2 then,

$\rho_1\,A_1\,v_1 = \rho_2\,A_2\,v_2$

so, this is the continuity equation for a compressible fluid

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13. Energy of a flowing fluid

There are following three types of energies in a flowing fluid.

(i) Pressure energy

if P is the pressure on the area A of a fluid, and the liquid moves through a distance due to this pressure, then pressure energy of liquid = work done

= force ×displacement

= PAl

The volume of the liquid is Al.

$\therefore$ Pressure energy per unit volume of liquid = $\frac{PAl}{Al}$ = P

(ii) Kinetic energy

If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy is $\frac{1}{2}$ mv^2

$\therefore$ kinetic energy per unit volume of liquid = $\frac{1}{2}\left(\frac{m}{v}\right)v^2 = \frac{1}{2}\rho v^2$

Here, ρ is the density of liquid.

(iii) Potential energy

If a liquid of mass m is at a height h from the reference line (h = 0), then its potential energy is mgh.

$\therefore$ Potential energy per unit volume of the liquid = $\left(\frac{m}{v}\right)gh = \rho gh$

14. Bernoulli's Equation

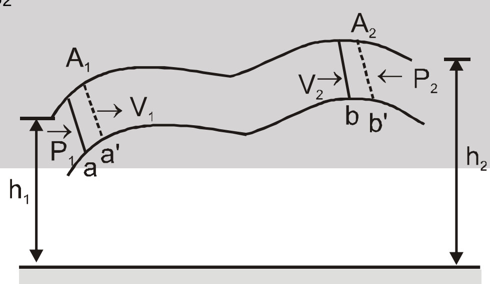
The Bernoulli's equation is **"Sum of total energy per unit volume (pressure + kinetic + potential) is constant for an Ideal fluid".**

$P + \frac{1}{2}\rho v^2 + \rho gh$ = constant (J/m^3)

Bernoulli's equation relates the pressure, flow speed and height for flow of an ideal (incompressible and nonviscous) fluid .The pressure of a fluid depends on height as the static situation, and it also depends on the speed of flow.

To derive Bernoulli's equation, we apply the work- energy theorem to the fluid in a section of the fluid element Consider the element of fluid that at some initial time lies between two cross sections a and b. The speeds at the lower and upper ends are v_1 and v_2 In a small time interval, the fluid that is initially at a moves to aa' distance aa' = $ds_1 = v_1$ dt and the fluid that is initially at b moves to b' distance bb' = $ds_2 = v_2dt$, The cross- section areas at the two ends are A_1 and A_2 as shown. The fluid is incompressible hence, by the continuity equation, the volume of fluid dV passing through and cross-section during time dt is the same.

That is, $dv = A_1\, ds_1 = A_2\, ds_2$



Work done on the Fluid Element

Let us calculate the work done on this element during interval dt. The pressure at the two ends are P_1 and P_2, the force on the cross section at a is $P_1 A_1$ and the force at b is $P_2 A_2$. The net work done dW on the element by the surrounding fluid during this displacement is,

$dW = P_1A_1\, ds_1 - P_2A_2\, ds_2 = (P_1 - P_2)dV$

Change in Potential Energy

At the beginning of dt the potential energy for the mass between a and a' is $dmgh_1 = \rho\ (dV)gh_1\ v_1^2$. At the end of dt the potential energy for the mass between b and b' is $(dm)gh_2 = \rho(dv)gh_2$ .The net change in potential energy dU during dt is ,

$$dU = \rho\ (dV)\ g\ (h_2 - h_1)$$

Change in Kinetic Energy

At the beginning of dt the fluid between a and a' has volume A_1ds_1, mass $\rho A_1\ ds_1$ and kinetic energy $\frac{1}{2}\ \rho(A_1\ ds_1)\ v_1^2$. At the end of dt the fluid between b and b' has kinetic energy $\frac{1}{2}\rho(A_2\ ds_2)\ v_2^2$. The net change in kinetic energy dK during time dt is.

$$dK = \frac{1}{2}\ \rho\ (dV)(\ v_2^2 - v_1^2\ )$$

Combining Eqs. (i),(ii) and (iii) in the energy equation,

$$dW = dK + dU$$

We obtain,

$$(P_1 - P_2)\ dV = \frac{1}{2}\rho dV\ (\ v_2^2 - v_1^2\ ) + \rho\ (h_2 - h_1\ )$$

or $$P_1 - P_2 = \frac{1}{2}\rho(\ v_2^2 - v_1^2\ ) + \rho g\ (h_2 - h_1)$$

This is Bernoulli's equation. It states that the work done on a unit Volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow. We can also express Eq. (iv) in a more convenient form as.

$$P_1 + \rho gh_1 + \frac{1}{2}\rho\ v_1^2\ = P_2 + \rho gh_2 + \frac{1}{2}\rho\ v_2^2$$

The subscripts 1and 2 refer to any two points along the flow tube, so we can also write

$$\rho + \rho gh + \rho v^2 = \text{constant}$$

Note: When the fluid is not moving ($v_1 = 0 = v_2$) Bernoulli' s equation reduces to ,

$$P_1 + \rho gh_1 = \rho_2 + \rho gh_2$$

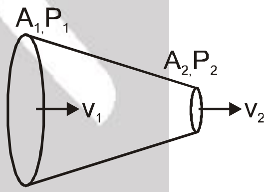
$\therefore$ $$P_1 - P_2 = \rho g(h_2 - h_1)$$

This is the pressure relation we derived for a fluid at rest.

Solved Examples

Example 19. Calculate the rate of flow of glycerine of density $1.25 \times 10^3\ kg/m^3$ through the conical section of a pipe, if the radii of its ends are 0.1m and 0.04 m and the pressure drop across its length is 10 N/m.

Solution : From continuity equation,



$$A_1\ v_1 = A_2\ v_2$$

or $$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right) = \left(\frac{0.04}{0.1}\right)^2 = \frac{4}{25}$$

From Bernoulli's equation ,

$$P_1 + \frac{1}{2}\rho\ v_1^2 = P_2 + \frac{1}{2}\rho\ v_2^2$$

or $$v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho}$$

or $$v_2^2 - v_1^2 = \frac{2 \times 10}{1.25 \times 10^3} = 1.6 \times 10^{-2}\ m^2/s$$

Solving Eqs. (i) and (ii), we get

$$v_2 \approx 0\ .128\ m/s$$

$\therefore$ Rate of volume flow through the tube

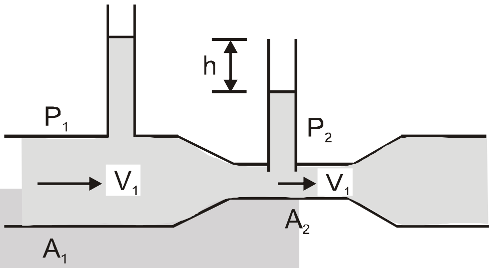
$$Q = A_2v_2 = \left(\pi r_2^2\right)v_2 = \pi\ (0.04)^2\ (0.128) = 6.43 \times 10^{-4}\ m^3/s$$

15. Applications Based on Bernoulli's Equation

(a) Venturimeter

Figure shows a venturimeter used to measure flow speed in a pipe of non- uniform cross-section. We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of the pipe, with $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho\, v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



From the continuity equation $v_2 = \frac{A_1 v_1}{A_2}$

Substituting and rearranging, we get

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right)$$

The pressure difference is also equal to ρgh, where h is the difference in liquid level in the two tubes. Substituting in Eq. (i) we get

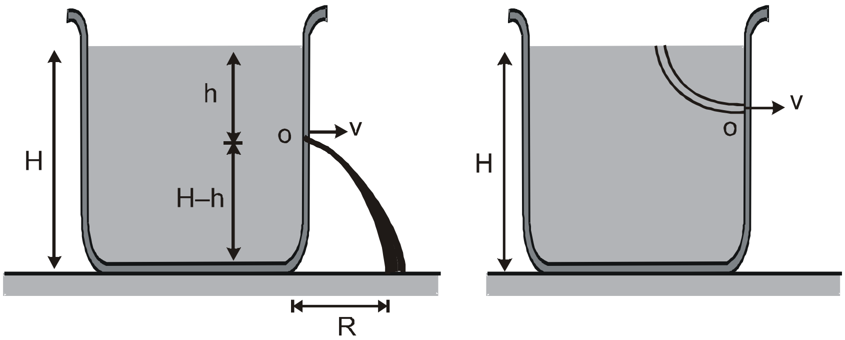
$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

The discharge or volume flow rate can be obtained as,

$$\frac{dV}{dt} = A_1 v_1 = A_1 \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

(b) Speed of Efflux

Suppose, the surface of a liquid in a tank is at a height h from the orifice O on its sides, through which the liquid issues out with velocity v. The speed of the liquid coming called the speed of efflux. If the dimensions of the tank be sufficiently large, the velocity of the liquid at its surface may be taken to be zero and since the pressure there as well as at the orifice O is the same viz atmospheric it plays no part in the flow of the liquid, which thus occurs purely in consequence of the hydrostatic pressure of the liquid itself. So that, considering a tube of flow, starting at the liquid surface and ending at the orifice, as shown in figure .Applying Bernoulli's equation we have



Total energy per unit volume of the liquid at the surface

= KE + PE + pressure energy = 0 + $\rho gh + P_0$

and total energy per unit volume at the orifice

$= KE + PE + pressure = \frac{1}{2}\rho v^2 + 0 + P_0$

Since total energy of the liquid must remain constant in steady flow, in accordance with Bernoulli's equation we have

$\rho gh + P_0 = \frac{1}{2}\rho v^2 + P_0$ or $v = \sqrt{2gh}$

Evangellista Torricelli showed that this velocity is the same as the liquid will attain in falling freely through the height (h) from the surface to the orifice. This is known as Torricelli's theorem and may be stated as. "The velocity of efflux of a liquid issuing out of an orifice is the same as it would attain if allowed to fall freely through the vertical height between the liquid surface and orifice.

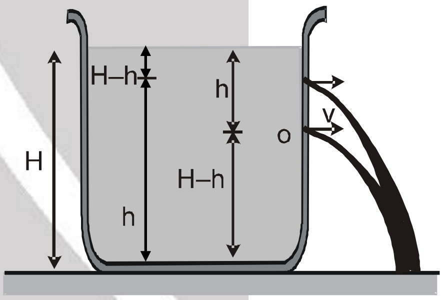
16. Range (R)

Let us find the range R on the ground.

Considering the vertical motion of the liquid, $(H - h) = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2(H-h)}{g}}$

Now, considering the horizontal motion, R = vt $R = \sqrt{2gh}\left(\sqrt{\frac{2(H-h)}{g}}\right)$ or $R = 2\sqrt{h(H-h)}$

From the expression of R, following conclusions can be drawn,



(i) $R_h = R_{H-h}$

as $R_h = 2\sqrt{h(H-h)}$ and $R_{H-h} = 2\sqrt{h(H-h)}$

This can be maximum at $h = \frac{H}{2}$ and $R_{max} = H$.

Proof : $R^2 = 4(Hh - h^2)$

For R to be maximum, $\frac{dR^2}{dh} = 0$

or $H - 2h = 0$ or $h = \frac{H}{2}$

That is, R is maximum at $h = \frac{H}{2}$ and $R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$

Time taken to empty a tank

We are here interested in finding the time required to empty a tank if a hole is made at the bottom of the tank.

Consider a tank filled with a liquid of density ρ upto a height H. A small hole of area of cross section a is made at the bottom of the tank. The area of cross-section of the tank is A.

Let at some instant of time the level of liquid in the tank is y. Velocity of efflux at this instant of time would be

$v = \sqrt{2gy}$

Now, at this instant volume of liquid coming out of the hole per second is $\left(\frac{dV_1}{dt}\right)$.

Volume of liquid coming down in the tank per second is $\left(\frac{dV_2}{dt}\right)$.

To calculate time taken to empty a tank $\frac{dV_1}{dt} = \frac{dV_2}{dt}$

$\therefore \quad av = A\left(-\frac{dy}{dt}\right) \quad \therefore \quad a\sqrt{2gy} = A\left(-\frac{dy}{dt}\right)$

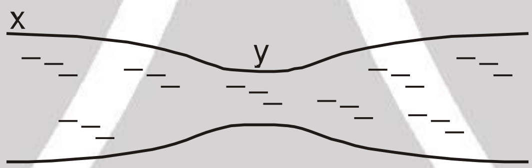
or $\quad \int_0^t dt = -\frac{A}{a\sqrt{2g}} \int_H^0 y^{-1/2}\ dy$

$\therefore \quad t = \frac{2A}{a\sqrt{2g}}\ [\sqrt{y}]_0^H$

$\therefore \quad t = \frac{A}{a}\sqrt{\frac{2H}{g}}$

Solved Examples

Example 20. Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between x and y where the areas of cross-section are 3cm^2 and 1.5cm^2 respectively. Find the rate of flow of water through the tube.



Solution : Let the velocity at x = v_x and that at y = v_y.

By the equation of continuity, $\frac{v_y}{v_x} = \frac{3cm^2}{1.5cm^2} = 2.$

By Bernoulli's equation,

$P_x + \frac{1}{2}\rho\, v_x^2 = P_y + \frac{1}{2}\rho v_y^2$

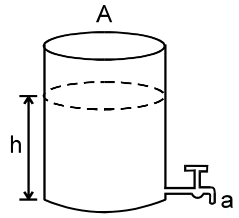
or, $P_x - P_y = \frac{1}{2}\rho(2v_y)^2 - \frac{1}{2}\rho v_y^2 = \frac{3}{2}\rho v_y^2$

or, $600\frac{N}{m^2} = \frac{3}{2}\left(1000\frac{kg}{m^3}\right)v_x^2$

or, $v_x = \sqrt{0.4m^2/s^2} = 0.63$ m/s.

The rate of flow = (3 cm^2) (0.63 m/s) = 189 cm^3/s.

Example 21. A cylindrical container of cross-section area, A is filled up with water upto height 'h'. Water may exit through a tap of cross section area 'a' in the bottom of container. Find out

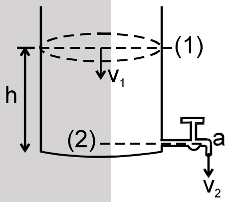


(a) Velocity of water just after opening of tap.

(b) The area of cross-section of water stream coming out of tape at depth h_0 below tap in terms of 'a' just after opening of tap.

(c) Time in which container becomes empty. (Given : $\left(\frac{a}{A}\right)^{1/2} = 0.02$, h = 20 cm, h_0 = 20 cm)

Solution : **(a)** Applying Bernoulli's equation between (1) and (2) -



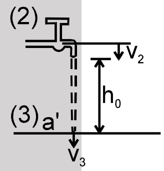
$$P_a + \rho gh + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$$

Through continuity equation :

$$Av_1 = av_2,\ v_1 = \frac{av_2}{a} \qquad \rho gh + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$

on solving - $v_2 = \sqrt{\dfrac{2gh}{1-\dfrac{a^2}{A^2}}}$ = 2m/sec. ....(1)

(b) Applying Bernoulli's equation between (2) and (3)



$$\frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho v_3^2$$

Through continuity equation –

$$av_2 = a'\,v_3 \ \Rightarrow \qquad v_3 = \frac{av_2}{a'}$$

$$\Rightarrow \ \frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho\left(\frac{av_2}{a'}\right)^2$$

$$\frac{1}{2} \times 2 \times 2 + gh_0 = \frac{1}{2}\left(\frac{a}{a'}\right)^2 \times 2 \times 2$$

$$\left(\frac{a}{a'}\right)^2 = 1 + \frac{9.8 \times .20}{2} \Rightarrow \left(\frac{a}{a'}\right)^2 = 1.98$$

$$\Rightarrow \ a' = \frac{a}{\sqrt{1.98}}$$

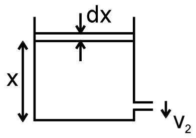
(c) From (1) at any height 'h' of liquid level in container, the velocity through tap,

$$v = \sqrt{\frac{2gh}{0.98}} = \sqrt{20h}$$

we know, volume of liquid coming out of tap = decrease in volume of liquid in container.

For any small time interval 'dt'

$$av_2dt = -\,A \cdot dx$$



$$a\sqrt{20x}\,dt = -A\,dx \Rightarrow \int_0^t dt = -\frac{A}{a}\int_h^0 \frac{dx}{\sqrt{20x}}$$

$$t = \frac{A}{a\sqrt{20}}\left[2\sqrt{x}\right]_h^0 \Rightarrow t = \frac{A}{a\sqrt{20}}\,2\sqrt{h}$$

$$= \frac{A}{a} \times 2 \times \sqrt{\frac{h}{20}} = \frac{2A}{a}\sqrt{\frac{0.20}{20}} = \frac{2A}{a} \times 0.1$$

Given $\left(\frac{a}{A}\right)^{1/2} = 0.02$ or $\frac{A}{a} = \frac{1}{0.0004} = 2500$

Thus t = 2 × 2500 × 0.1 = **500 second**.

Example 22. A tank is filled with a liquid upto a height H .A small hole is made at the bottom of this tank Let t_1 be the time taken to empty first half of the tank and t_2 is the time taken to empty rest half of the tank then find $\frac{t_1}{t_2}$.

Solution : Substituting the proper limits in Eq. (i), derived in the theory, we have

$$\int_0^{t_1} dt = -\frac{A}{a\sqrt{2g}}\int_H^{H/2} y^{-1/2}dy$$

or $t_1 = \frac{2A}{a\sqrt{2g}}\,[\sqrt{y}]_{H/2}^{H}$ or $t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$ or $t_1 = \frac{A}{a}\sqrt{\frac{H}{g}}\left(\sqrt{2}-1\right)$

Similarly $\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_{H/2}^{0} y^{-1/2} = dy$

or $t_2 = \frac{A}{a}\sqrt{\frac{H}{g}}$

We get

$\frac{t_1}{t_2} = \sqrt{2} - 1$ or $\frac{t_1}{t_2} = 0.414$

Note : From here we see that $t_1 < t_2$ This is because initially the pressure is high and the liquid comes out with

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

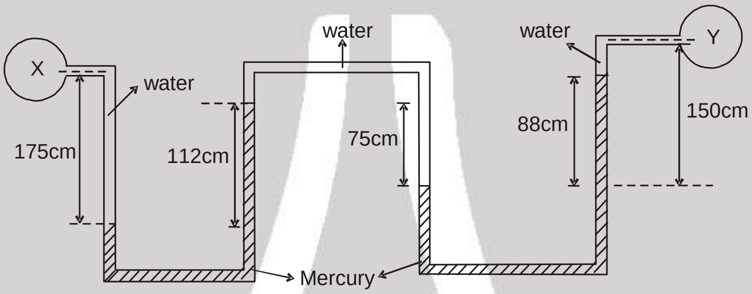
Section (A) : Measurement and calculation of pressure

A-1. We can cut an apple easily with a sharp knife as compared to a blunt knife. Explain why?

A-2. Why mercury is used in barometers instead of water ?

A-3. Pressure 3 m below the free surface of a liquid is $15 KN/m^2$ in excess of atmosphere pressure. Determine its density and specific gravity. [g = 10 m/sec^2]

A-4.✍ Two U-tube manometers are connected to a same tube as shown in figure. Determine difference of pressure between X and Y. Take specific gravity of mercury as 13.6. (g = 10 m/s^2, ρ_{Hg} = 13600 kg/m^3)



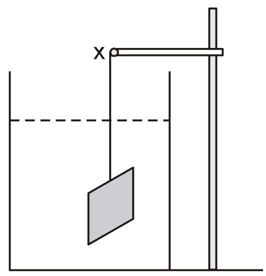
A-5. A rectangular vessel is filled with water and oil in equal proportion (by volume), the oil being twice lighter than water. Show that the force on each side wall of the vessel will be reduced by one fifth if the vessel is filled only with oil. (Assume atmospheric pressure is negligible)

Section (B) : Archemedies principle and force of buoyancy

B-1. A cube of wood supporting a 200 gm mass just floats in water. When the mass is removed the cube rises by 2 cm at equilibrium. Find side of the cube.

B-2.✍ A small solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enters into water. Upto what depth will the ball go ? How much time will it take to come again to the water surface? Neglect air resistance, viscosity effects of water and energy loss due to collision at water surface. (g = 9.8 m/s^2)

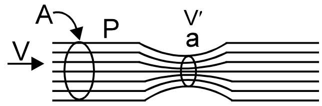
B-3. A metallic square plate is suspended from a point x as shown in figure. The plate is made to dip in water such that level of water is well above that of the plate. The point ‘x’ is then slowely raised at constant velocity. Sketch the variation of tension T in string with the displacement ‘s’ of point x.



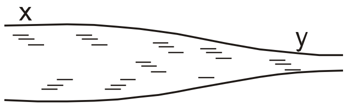
Section (C) : Continuity equation & Bernoulli theorem and their application

C-1. Calculate the rate of flow of glycerin of density 1.25 x10^3 kg/m^3 through the conical section of a pipe placed horizontally, if the radii of its ends are 0.1m and 0.04 m and the pressure drop across its length is 10 N/m^2.

C-2. Consider the Venturi tube of Figure. Let area A equal 5a. Suppose the pressure at A is 2.0 atm. Compute the values of velocity v at 'A' and velocity v′ at 'a' that would make the pressure p′ at 'a' equal to zero. Compute the corresponding volume flow rate if the diameter at A is 5.0 cm. (The phenomenon at a when p′ falls to nearly zero is known as cavitation. The water vaporizes into small bubbles.) (P_{atm} = 10^5 N/m^2, ρ = 1000 kg/m^3).



C-3. Water flows through a horizontal tube of variable cross-section (figure). The area of cross-section at x and y are 40 mm^2 and 20 mm^2 respectively. If 10 cc of water enters per second through x, find (i) the speed of water at x, (ii) the speed of water at y and (iii) the pressure difference $P_x - P_y$.



C-4. Suppose the tube in the previous problem is kept vertical with x upward but the other conditions remain the same. The separation between the cross-section at x and y is 15/16 cm. Repeat parts (i), (ii) and (iii) of the previous problem. Take g = 10 m/s^2.

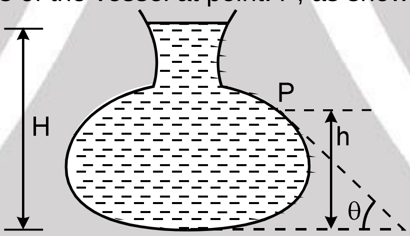
C-5. Suppose the tube in the previous problem is kept vertical with y upward. Water enters through y at the rate of 10 cm^3/s. Repeat part (iii). Note that the speed decreases as the water falls down.

C-6. Let air be at rest at the front edge of wing of an aeroplane and air passing over the surface of the wing at a fast speed v. If density of air is ρ, then find out the highest value for v in stream line flow when atmospheric pressure is p_{atm}.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Measurement and calculation of pressure

A-1. Figure here shows the vertical cross-section of a vessel filled with a liquid of density ρ. The normal thrust per unit area on the walls of the vessel at point. P, as shown, will be



(A) $h\rho g$ (B) $H\rho g$ (C) $(H - h)\rho g$ (D) $(H - h)\rho g \cos\theta$

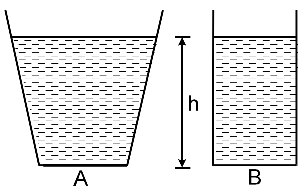
A-2. A tank with length 10 m, breadth 8 m and depth 6m is filled with water to the top. If g = 10 m s^{-2} and density of water is 1000 kg m^{-3}, then the thrust on the bottom is (neglect atmospheric pressure)

(A) 6 × 1000 × 10 × 80 N (B) 3 × 1000 × 10 × 48 N
(C) 3 × 1000 × 10 × 60 N (D) 3 × 1000 × 10 × 80 N

A-3. In a hydraulic lift, used at a service station the radius of the large and small piston are in the ratio of 20 : 1. What weight placed on the small piston will be sufficient to lift a car of mass 1500 kg ?

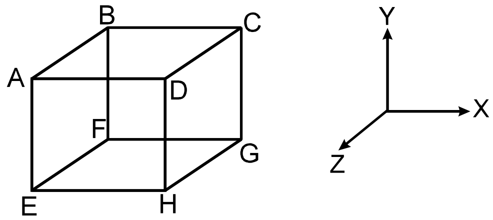
(A) 3.75 kg (B) 37.5 kg (C) 7.5 kg (D) 75 kg.

A-4. Two vessels A and B of different shapes have the same base area and are filled with water up to the same height h (see figure). The force exerted by water on the base is F_A for vessel A and F_B for vessel B. The respective weights of the water filled in vessels are W_A and W_B. Then



(A) $F_A > F_B$; $W_A > W_B$ (B) $F_A = F_B$; $W_A > W_B$ (C) $F_A = F_B$; $W_A < W_B$ (D) $F_A > F_B$; $W_A = W_B$

A-5.(i) The cubical container ABCDEFGH which is completely filled with an ideal (nonviscous and incompressible) fluid, moves in a gravity free space with a acceleration of $a = a_0(\hat{i} - \hat{j} + \hat{k})$ where a_0 is a positive constant. Then the only point in the container shown in the figure where pressure is maximum, is



(A) B (B) C (C) E (D) F

(ii) In previous question pressure will be minimum at point –

(A) A (B) B (C) H (D) F

Section (B) : Archemedies principle and force of buoyancy

B-1. The density of ice is x gm/cc and that of water is y gm/cc. What is the change in volume in cc, when m gm of ice melts?

(A) M (y – x) (B) (y – x)/m (C) mxy (x – y) (D) m (1/y – 1/x)

B-2. The reading of a spring balance when a block is suspended from it in air is 60 newton. This reading is changed to 40 newton when the block is fully submerged in water. The specific gravity of the block must be therefore :

(A) 3 (B) 2 (C) 6 (D) 3/2

B-3. A block of volume V and of density σ_b is placed in liquid of density $\sigma_l(\sigma_l > \sigma_b)$, then block is moved upward upto a height h and it is still in liquid. The increase in gravitational potential energy of the block is :

(A) $\sigma_b Vgh$ (B) $(\sigma_b + \sigma_l)Vgh$ (C) $(\sigma_b - \sigma_l)Vgh$ (D) none of these

B-4. A block of steel of size 5 cm × 5 cm × 5 cm is weighed in water. If the relative density of steel is 7. Its apparent weight is :

(A) 6 × 5 × 5 × 5 gf (B) 4 × 4 × 4 × 7 gf (C) 5 × 5 × 5 × 7 gf (D) 4 × 4 × 4 × 6 gf

B-5. A metallic sphere floats in an immiscible mixture of water ($\rho_w = 10^3$ kg/m^3) and a liquid ($\rho_L = 13.5 \times 10^3$) with (1/5)th portion by volume in the liquid and remaining in water. The density of the metal is :

(A) 4.5×10^3 kg/m^3 (B) 4.0×10^3 kg/m^3 (C) 3.5×10^3 kg/m^3 (D) 1.9×10^3 kg/m^3

B-6. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 36 g and its density is 9 g/cc. If the mass of the other is 48 g, its density in g/cc is :

(A) 4/3 (B) 3/2 (C) 3 (D) 5

B-7. In order that a floating object be in a stable equilibrium, its centre of buoyancy should be

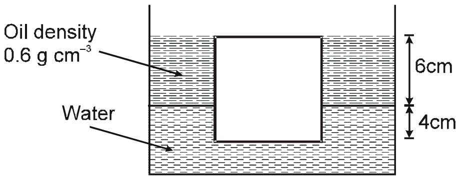
(A) vertically above its centre of gravity

(B) vertically below its centre of gravity

(C) horizontally in line with its centre of gravity

(D) may be anywhere

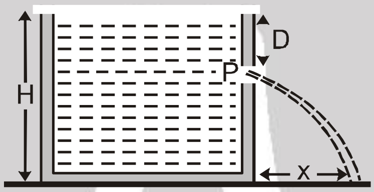
B-8. A cubical block of wood 10 cm on a side, floats at the interface of oil and water as shown in figure. The density of oil is 0.6 g cm^{-3} and density of water is 1 g cm^{-3}. The mass of the block is



(A) 706 g (B) 607 g (C) 760 g (D) 670 g

Section (C) : Continuity equation and Bernoulli theorem & their application

C-1. A tank is filled with water up to height H. Water is allowed to come out of a hole P in one of the walls at a depth D below the surface of water as shown in the figure. Express the horizontal distance x in terms of H and D :

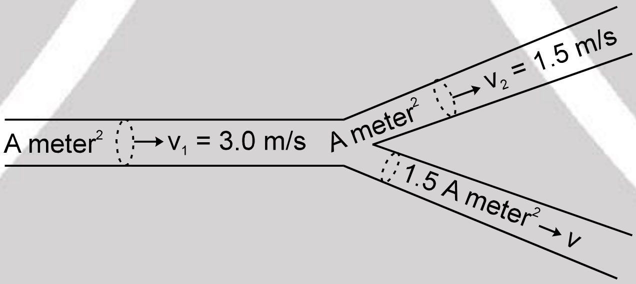


(A) $x = \sqrt{D(H-D)}$ (B) $x = \sqrt{\frac{D(H-D)}{2}}$ (C) $x = 2\sqrt{D(H-D)}$ (D) $x = 4\sqrt{D(H-D)}$

C-2. A fixed cylindrical vessel is filled with water up to height H. A hole is bored in the wall at a depth h from the free surface of water. For maximum horizontal range h is equal to :

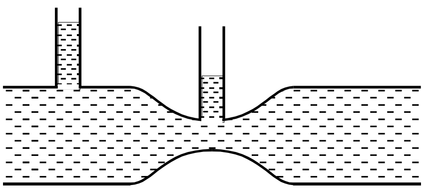
(A) H (B) 3H/4 (C) H/2 (D) H/4

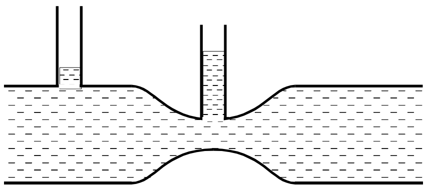
C-3. An incompressible liquid flows through a horizontal tube as shown in the figure. Then the velocity ' v ' of the fluid is :

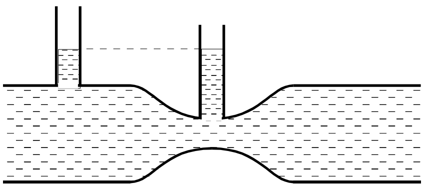


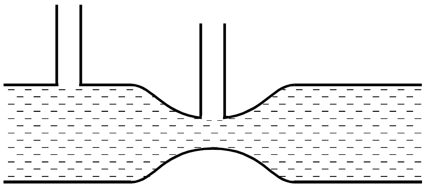
(A) 3.0 m/s (B) 1.5 m/s (C) 1.0 m/s (D) 2.25 m/s

C-4. For a fluid which is flowing steadily in a horizontal tube as shown in the figure, the level in the vertical tubes is best represented by

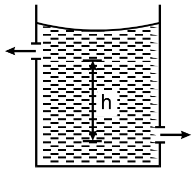
(A) 

(B) 

(C) 

(D) 

C-5. There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in height of the two holes is h as shown in the figure. As the liquid comes out of the two holes, the tank will experience a net horizontal force proportional to:



(A) $h^{1/2}$ (B) h (C) $h^{3/2}$ (D) h^2

C-6. A cylindrical tank of height 0.4 m is open at the top and has a diameter 0.16 m. Water is filled in it up to a height of 0.16 m. How long it will take to empty the tank through a hole of radius 5×10^{-3} m at its bottom ?

(A) 46.26 sec. (B) 4.6 sec. (C) 462.6 sec. (D) 0.46 sec.

C-7. A large cylindrical vessel contains water to a height of 10m. It is found that the thrust acting on the curved surface is equal to that at the bottom. If atmospheric pressure can support a water column of 10m, the radius of the vessel is **[Olympiad 2014 (stage-1)]**

(A) 10 m (B) 15m (C) 5m (D) 25m

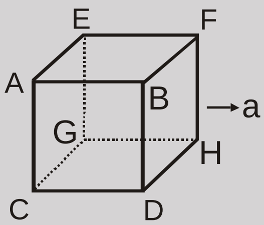
C-8. A jet of water of cross-sectional area A hits a plate normally with velocity v. the plate is moving in the direction of the jet with velocity V. therefore, the force exerted on the plate is proportional to

[Olympiad 2015 (stage-1)]

(A) v (B) v^2 (C) (v – V) (D) $(v – V)^2$

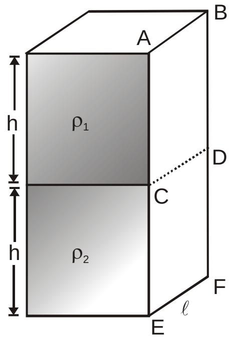
PART - III : MATCH THE COLUMN

1. A cubical box is completely filled with mass m of a liquid and is given horizontal acceleration a as shown in the figure. Match the force due to fluid pressure on the faces of the cube with their appropriate values (assume zero pressure as minimum pressure)



Column I	**Column II**
(A) force on face ABFE	(p) $\frac{ma}{2}$
(B) force on face BFHD	(q) $\frac{mg}{2}$
(C) force on face ACGE	(r) $\frac{ma}{2}+\frac{mg}{2}$
(D) force on face CGHD	(s) $\frac{ma}{2}$ +mg
	(t) $\frac{mg}{2}$ + ma

2. A cuboid is filled with liquid of density ρ_2 upto height h & with liquid of density ρ_1, also upto height h as shown in the figure



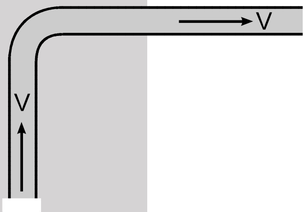
Column I	Column II
(A) Force on face ABCD due to liquid of density ρ_1	(p) zero
(B) Force on face ABCD due to liquid of density ρ_2	(q) $\frac{\rho_1 gh^2\ell}{2}$
(C) Force on face CDEF transferrred due to liquid of density ρ_1	(r) $\rho_1 gh^2\ell$
(D) Force on face CDEF due to liquid of density ρ_2 only	(s) $\frac{\rho_2 gh^2\ell}{2}$

Exercise-2

Marked Questions can be used as Revision Questions.

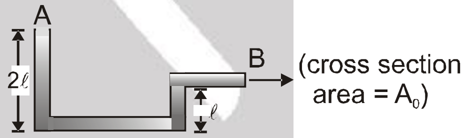
PART - I : ONLY ONE OPTION CORRECT TYPE

1. A fire hydrant (as shown in the figure) delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is



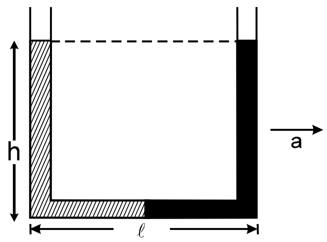
(A) ρVL (B) zero (C) 2ρVL (D) $\sqrt{2}\rho$VL

2. A tube in vertical plane is shown in figure. It is filled with a liquid of density ρ and its end B is closed. Then the force exerted by the fluid on the tube at end B will be : [Neglect atmospheric pressure and assume the radius of the tube to be negligible in comparison to ℓ]



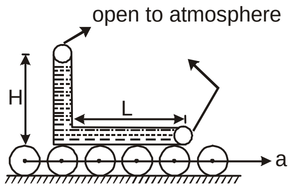
(A) 0 (B) $\rho g\ell A_0$ (C) $2\rho g\ell A_0$ (D) $\frac{\rho g\ell A_0}{2}$

3. A U-tube of base length "ℓ" filled with same volume of two liquids of densities ρ and 2ρ is moving with an acceleration "a" on the horizontal plane as shown in the figure. If the height difference between the two surfaces (open to atmosphere) becomes zero, then the height h is given by:



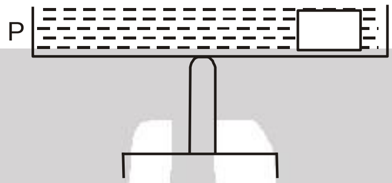
(A) $\frac{a}{2g}\ell$ (B) $\frac{3a}{2g}\ell$ (C) $\frac{a}{g}\ell$ (D) $\frac{2a}{3g}\ell$

4. A narrow tube completely filled with a liquid is lying on a series of cylinders as shown in figure. Assuming no sliding between any surfaces, the value of acceleration of the cylinders for which liquid will not come out of the tube from anywhere is given by



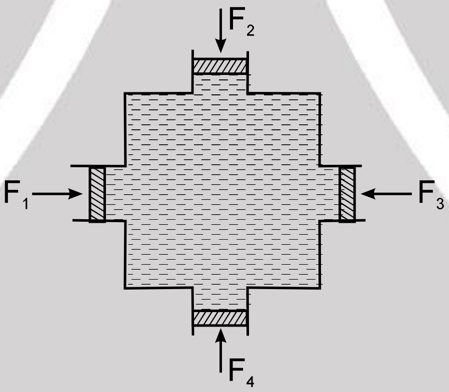
(A) $\frac{gH}{2L}$ (B) $\frac{gH}{L}$ (C) $\frac{2gH}{L}$ (D) $\frac{gH}{\sqrt{2}L}$

5. An open pan P filled with water (density ρ_w) is placed on a vertical rod, maintaining equilibrium. A block of density ρ is placed on one side of the pan as shown in the figure. Water depth is more than height of the block.



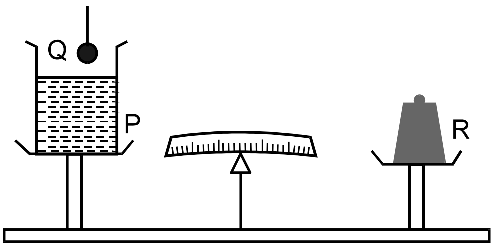
(A) Equilibrium will be maintained only if $\rho < \rho_w$.
(B) Equilibrium will be maintained only if $\rho \le \rho_w$.
(C) Equilibrium will be maintained for all relations between ρ and ρ_w.
(D) It is not possible to maintained the equilibrium

6. In the figure shown water is filled in a symmetrical container. Four pistons of equal area A are used at the four opening to keep the water in equilibrium. Now an additional force F is applied at each piston. The increase in the pressure at the centre of the container due to this addition is



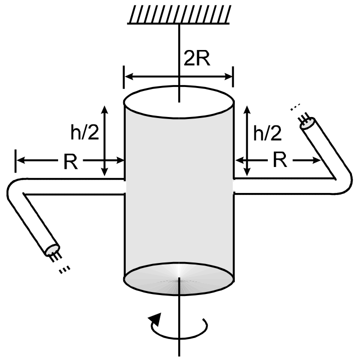
(A) $\frac{F}{A}$ (B) $\frac{2F}{A}$ (C) $\frac{4F}{A}$ (D) 0

7. Figure shows a weighing-bridge, with a beaker P with water on one pan and a balancing weight R on the other. A solid ball Q is hanging with a thread outside water. It has volume 40 cm^3 and weighs 80 g. If this solid is lowered to sink fully in water, but not touching the beaker anywhere, the balancing weight R' will be



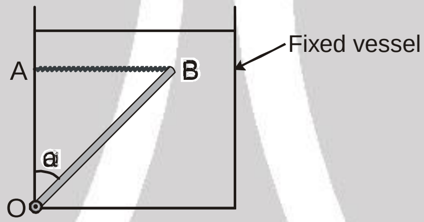
(A) same as R (B) 40 g less than R (C) 40 g more than R (D) 80 g more than R

8. A cylindrical container of radius ' R ' and height ' h ' is completely filled with a liquid. Two horizontal L shaped pipes of small cross-section area ' a ' are connected to the cylinder as shown in the figure. Now the two pipes are opened and fluid starts coming out of the pipes horizontally in opposite directions. Then the torque due to ejected liquid on the system is:



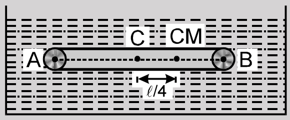
(A) 4aghρR (B) 8aghρR
(C) 2aghρR (D) aghρR

9. A uniform rod OB of length 1m, cross-sectional area 0.012 m^2 and relative density 2.0 is free to rotate about O in vertical plane. The rod is held with a horizontal string AB which can withstand a maximum tension of 45 N. The rod and string system is kept in water as shown in figure. The maximum value of angle α which the rod can make with vertical without breaking the string is



(A) 45º (B) 37º (C) 53º (D) 60º

10. A non uniform cylinder of mass m, length ℓ and radius r is having its centre of mass at a distance $\ell/4$ from the centre and lying on the axis of the cylinder as shown in the figure. The cylinder is kept in a liquid of uniform density ρ. The moment of inertia of the rod about the centre of mass is I. The angular acceleration of point A relative to point B just after the rod is released from the position shown in figure is

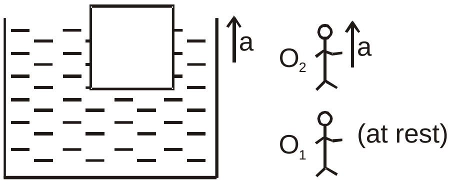


(A) $\frac{\pi\rho g\ell^2 r^2}{I}$ (B) $\frac{\pi\rho g\ell^2 r^2}{4I}$ (C) $\frac{\pi\rho g\ell^2 r^2}{2I}$ (D) $\frac{3\pi\rho g\ell^2 r^2}{4I}$

11. A block of iron is kept at the bottom of a bucket full of water at 2°C. The water exerts buoyant force on the block. If the temperature of water is increased by 1°C the temperature of iron block also increases by 1°C. The buoyant force on the block by water
(A) will increase (B) will decrease (C) will not change
(D) may decrease or increase depending on the values of their coefficient of expansion

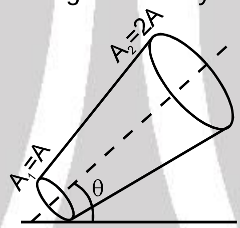
12. A liquid is kept in a cylindrical vessel which is rotated about its axis. The liquid rises at the sides. If the radius of the vessel is 0.05 m and the speed of rotation is 2 rev/s. The difference in the height of the liquid at the centre of the vessel and its sides will be (π^2 = 10) :
(A) 3 cm (B) 2 cm (C) 3/2 cm (D) 2/3 cm

13. A block is partially immersed in a liquid and the vessel is accelerating upwards with an acceleration "a". The block is observed by two observers O_1 and O_2 , one at rest and the other accelerating with an acceleration "a" upward as shown in the figure. The total buoyant force on the block is :



(A) same for O_1 and O_2
(B) greater for O_1 than O_2
(C) greater for O_2 than O_1
(D) data is not sufficient

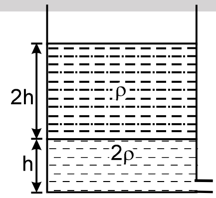
14. A portion of a tube is shown in the figure. Fluid is flowing from cross-section area A_1 to A_2. The two cross-sections are at distance 'ℓ' from each other. The velocity of the fluid at section A_2 is $\sqrt{\frac{g\ell}{2}}$. If the pressures at A_1 & A_2 are same, then the angle made by the tube with the horizontal will be:



(A) 37º
(B) $\sin^{-1}\frac{3}{4}$
(C) 53º
(D) $\cos^{-1}\frac{3}{4}$

15. There is a small hole in the bottom of a fixed container containing a liquid upto height 'h'. The top of the liquid as well as the hole at the bottom are exposed to atmosphere. As the liquid comes out of the hole. (Area of the hole is 'a' and that of the top surface is 'A') :
(A) the top surface of the liquid accelerates with acceleration $= g$
(B) the top surface of the liquid accelerates with acceleration $= g\frac{a^2}{A^2}$
(C) the top surface of the liquid retards with retardation $= g\frac{a}{A}$
(D) the top surface of the liquid retards with retardation $= \frac{ga^2}{A^2}$

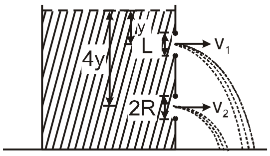
16. The velocity of the liquid coming out of a small hole of a large vessel containing two different liquids of densities 2ρ and ρ as shown in figure is



(A) $\sqrt{6gh}$
(B) $2\sqrt{gh}$
(C) $2\sqrt{2gh}$
(D) $\sqrt{gh}$

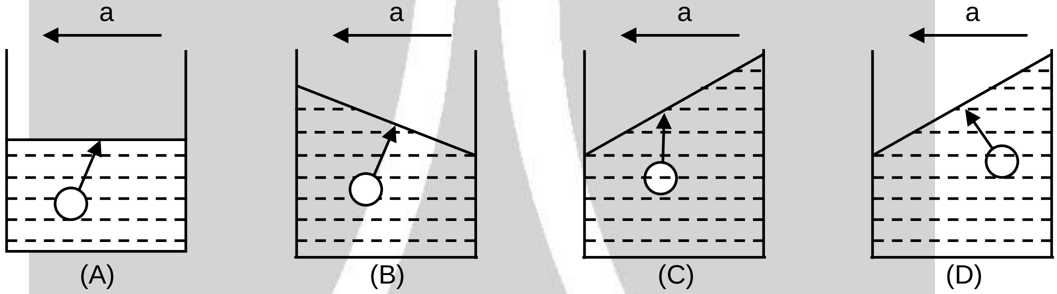
17. Two water pipes P and Q having diameters 2×10^{-2} m and 4×10^{-2} m, respectively, are joined in series with the main supply line of water. The velocity of water flowing in pipe P is
(A) 4 times that of Q
(B) 2 times that of Q
(C) 1/2 times that of Q
(D) 1/4 times that of Q

18. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then radius R, is equal to : **[JEE - 2000, 2/105]**



(A) $\frac{L}{\sqrt{2\pi}}$ (B) $2\pi L$ (C) L (D) $\frac{L}{2\pi}$

19. A cup of water is placed in a car moving at a constant acceleration a to the left. Inside the water is a small air bubble. The figure that correctly shows the shape of the water surface and the direction of motion of the air bubble is. **[Olympiad (State-1) 2016]**



(A) A (B) B (C) C (D) D

20. Two identical solid block A and B are made of two different materials. Block A floats in a liquid with half of its volume submerged. When block B is pasted over A, the combination is found to just float in the liquid. The ratio of the densities of the liquid, material of A and material of B is given by **[Olympiad (Stage-1) 2017]**

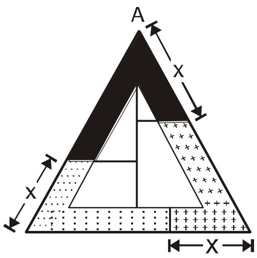
(A) 1 : 2 : 3 (B) 2 : 1 : 4 (C) 2 : 1 : 3 (D) 1 : 3 : 2

21. A hollow sphere of inner radius 9 cm and outer radius 10 cm floats half submerged in a liquid of specific gravity 0.8. The density of the material of the spere is **[Olympiad (Stage-1) 2017]**

(A) 0.84g cm^{-3} (B) 1. 48g cm^{-3} (C) 1. 84g cm^{-3} (D) 1.24g cm^{-3}

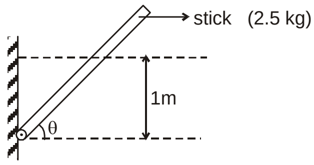
PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A closed tube in the form of an equilateral triangle of side ℓ = 3m contains equal volumes of three liquids which do not mix and is placed vertically with its lowest side horizontal. Find 'x' (in meter) in the figure if the densities of the liquids are in A.P.

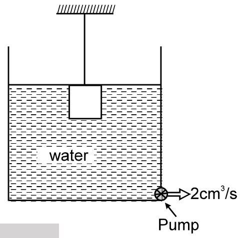


2. An open tank 10 m long and 2m deep is filled upto height 1.5 m of oil of specific gravity 0.80. The tank is accelerated uniformly from rest to a speed of 10 m/sec. The shortest time (in seconds) in which this speed may be attained without spilling any oil (in sec). [g = $10 m/s^2$]

3. A stick of square cross-section (5 cm × 5 cm) and length '4m' weighs 2.5 kg is in equilibrium as shown in the figure below. Determine its angle of inclination (in degree) in equilibrium when the water surface is 1 m above the hinge.

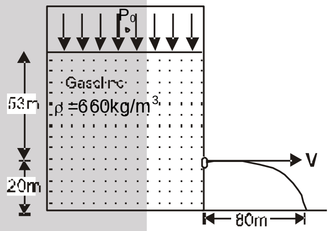


4. Figure shows a cubical block of side 10 cm and relative density 1.5 suspended by a wire of cross sectional area 10^{-6} m^2. The breaking stress of the wire is 7×10^6 N/m^2. The block is placed in a beaker of base area 200 cm^2 and initially i.e. at t = 0, the top surface of water & the block coincide. There is a pump at the bottom corner which ejects 2 cm^3 of water per sec constantly. If the time at which the wire will break is $(20)\alpha$ (in second) then find 'α' .



5. A cylindrical vessel filled with water upto a height of 2m stands on horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. If the minimum diameter of the hole so that the vessel begins to move on the floor if the plug is removed is $\frac{x}{10\sqrt{\pi}}$ meter then x will be (if the coefficient of friction between the bottom of the vessel and the plane is 0.4 and total mass of water plus vessel is 100 kg.)

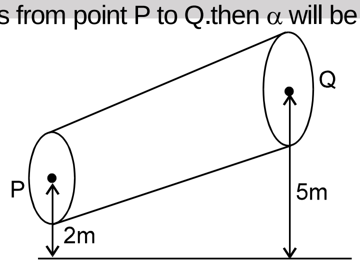
6. A tank containing gasoline is sealed and the gasoline is under pressure P_0 as shown in the figure. The stored gasoline has a density of 660 kg m^{-3}. A sniper fires a rifle bullet into the gasoline tank, making a small hole 53 m below the surface of gasoline. The total height of gasoline is 73 m from the base. The jet of gasoline shooting out of the hole strikes the ground at a distance of 80 m from the tank initially. If the pressure above the gasoline surface is $(1.39)\alpha \times 10^5$ N/m^2 than α is- (The local atmospheric pressure is 10^5 Nm^{-2})



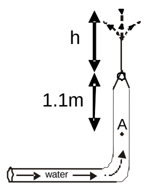
7. A large open top container of negligible mass and uniform cross–sectional area A has a small hole of cross–sectional area $\frac{A}{100}$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at t = 0, The acceleration of the container is $\frac{x}{10}$ m/s^2 than x is.

[JEE - 1997 Cancel, 5/100]

8. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross-section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. If the work done per unit volume by the pressure is $(1161)\,\alpha$ joule/m^3 as the liquid flows from point P to Q.then α will be (g = 9.8 m/s^2) **[JEE - 1997, 5/100]**



9. Water shoots out of a pipe and nozzle as shown in the figure. The cross–sectional area for the tube at point A is four times that of the nozzle. The pressure of water at point A is 41×10^3 Nm^{-2} (guage). If the height 'h' above the nozzle to which water jet will shoot is x/10 meter than x is. (Neglect all the losses occurred in the above process) [g = 10 m/s^2]

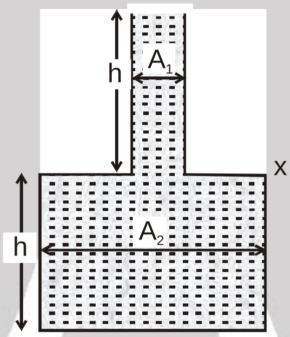


PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. An air bubble in a water tank rises from the bottom to the top. Which of the following statements are true ?
(A) Bubble rises upwards because pressure at the bottom is less than that at the top.
(B) Bubble rises upwards because pressure at the bottom is greater than that at the top.
(C) As the bubble rises, its size increases.
(D) As the bubble rises, its size decreases.

2. Pressure gradient in a static fluid is represented by (z–direction is vertically upwards, and x-axis is along horizontal, d is density of fluid) :
(A) $\frac{\partial p}{\partial z} = -dg$ (B) $\frac{\partial p}{\partial x} = dg$ (C) $\frac{\partial p}{\partial x} = 0$ (D) $\frac{\partial p}{\partial z} = 0$

3. The vessel shown in Figure has two sections of area of cross-section A_1 and A_2. A liquid of density ρ fills both the sections, up to height h in each. Neglecting atomospheric pressure,

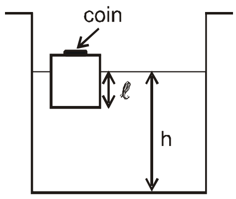


(A) the pressure at the base of the vesel is $2h\rho g$
(B) the weight of the liquid in vessel is equal to $2h\rho gA_2$
(C) the force exerted by the liquid on the base of vessel is $2h\rho gA_2$
(D) the walls of the vessel at the level X exert a force $h\rho g(A_2 - A_1)$ downwards on the liquid.

4. A cubical block of wood of edge 10cm and mass 0.92kg floats on a tank of water with oil of rel. density 0.6. Thickness of oil is 4cm above water. When the block attains equilibrium with four of its sides edges vertical:
(A) 1 cm of it will be above the free surface of oil.
(B) 5 cm of it will be under water.
(C) 2 cm of it will be above the common surface of oil and water.
(D) 8 cm of it will be under water.

5. Following are some statements about buoyant force, select incorrect statement/statements (Liquid is of uniform density)
(A) Buoyant force depends upon orientation of the concerned body inside the liquid.
(B) Buoyant force depends upon the density of the body immersed.
(C) Buoyant force depends on the fact whether the system is on moon or on the earth.
(D) Buoyant force depends upon the depth at which the body (fully immersed in the liquid) is placed inside the liquid.

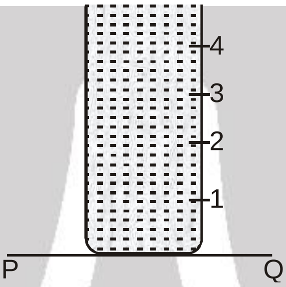
6. A wooden block with a coin placed on its top, floats in water as shown in figure. The distance ℓ and h are shown here. After some time the coin falls into the water. Then : **[JEE-2002 (Screening), 3/105]**



(A) ℓ decreases and h increase (B) ℓ increases and h decreases
(C) both ℓ and h increases (D) both ℓ and h decrease

7. A block of density 2000 kg/m^3 and mass 10 kg is suspended by a spring stiffness 100 N/m. The other end of the spring is attached to a fixed support. The block is completely submerged in a liquid of density 1000 kg/m^3 If the block is in equilibrium position then,
(A) the elongation of the spring is 1 cm.
(B) the magnitude of buoyant force acting on the block is 50 N.
(C) the spring potential energy is 12.5 J.
(D) magnitude of spring force on the block is greater than the weight of the block.

8. A cylindrical vessel of 90 cm height is kept filled upto the brim as shown in the figure. It has four holes 1, 2, 3, 4 which are respectively at heights of 20cm, 30 cm, 40 cm and 50 cm from the horizontal floor PQ. The water falling at the maximum horizontal distance from the vessel comes from

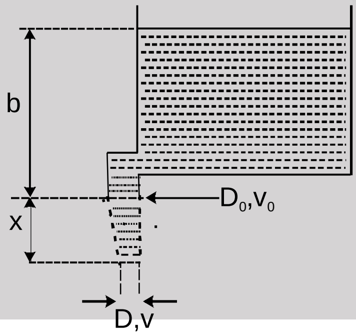


(A) hole number 4 (B) hole number 3 (C) hole number 2 (D) hole number 1.

PART - IV : COMPREHENSION

Comprehension-1

The figure shows the commonly observed decrease in diameter of a water stream as it falls from a tap. The tap has internal diameter D_0 and is connected to a large tank of water. The surface of the water is at a height b above the end of the tap.
By considering the dynamics of a thin "cylinder" of water in the stream answer the following: (Ignore any resistance to the flow and any effects of surface tension, given ρ_w = density of water)



1. Equation for the flow rate, i.e. the mass of water flowing through a given point in the stream per unit time, as function of the water speed v will be
(A) $v \rho_w \pi D^2 / 4$ (B) $v\rho_w(\pi D^2/4 - \pi D_0^2/4)$ (C) $v \rho_w \pi D^2 / 2$ (D) $v \rho_w \pi D_0^2 / 4$

2. Which of the following equation expresses the fact that the flow rate at the tap is the same as at the stream point with diameter D and velocity v (i.e. D in terms of D_0, v_0 and v will be) :
(A) $D = \frac{D_0 v_0}{v}$ (B) $D = \frac{D_0 v_0^2}{v^2}$ (C) $D = \frac{D_0 v}{v_0}$ (D) $D = D_0\sqrt{\frac{v_0}{v}}$

3. The equation for the water speed v as a function of the distance x below the tap will be :
(A) $v = \sqrt{2gb}$ (B) $v = [2g(b + x)]^{1/2}$ (C) $v = \sqrt{2gx}$ (D) $v = [2g(b - x)]^{1/2}$

4. Equation for the stream diameter D in terms of x and D_0 will be :

(A) $D = D_0\left(\frac{b}{b+x}\right)^{1/4}$ (B) $D = D_0\left(\frac{b}{b+x}\right)^{1/2}$ (C) $D = D_0\left(\frac{b}{b+x}\right)$ (D) $D = D_0\left(\frac{b}{b+x}\right)^{2}$

5. A student observes after setting up this experiment that for a tap with D_0 = 1 cm at x = 0.3 m the stream diameter D = 0.9 cm. The heights b of the water above the tap in this case will be :

(A) 5.7 cm (B) 57 cm (C) 27 cm (D) 2.7 cm

Comprehension-2

One way of measuring a person's body fat content is by "weighing" them under water. This works because fat tends to float on water as it is less dense than water. On the other hand muscle and bone tend to sink as they are more dense. Knowing your "weight" under water as well as your real weight out of water, the percentage of your body's volume that is made up of fat can easily be estimated. This is only an estimate since it assumes that your body is made up of only two substances, fat (low density) and everything else (high density). The "weight" is measured by spring balance both inside and outside the water. Quotes are placed around weight to indicate that the measurement read on the scale is not your true weight, i.e. the force applied to your body by gravity, but a measurement of the net downward force on the scale.

6. Ram and Shyam are having the same weight when measured outside the water. When measured under water, it is found that weight of Ram is more than that of Shyam, then we can say that

(A) Ram is having more fat content than Shyam.

(B) Shyam is having more fat content that Ram.

(C) Ram and Shyam both are having the same fat content.

(D) None of these.

7. Ram is being weighed by the spring balance in two different situations. First when he was fully immersed in water and the second time when he was partially immersed in water, then

(A) Reading will be more in the first case. (B) Reading will be more in the second case.

(C) Reading would be same in both the cases. (D) Reading will depend upon experimental setup.

8. Salt water is denser than fresh water. If Ram is immersed fully first in salt water and then in fresh water and weighed, then

(A) Reading would be less in salt water.

(B) Reading would be more in salt water.

(C) Reading would be the same in both the cases.

(D) reading could be less or more.

9. A person of mass 165 Kg having one fourth of his volume consisting of fat (relative density 0.4) and rest of the volume consisting of everything else (average relative density $\frac{4}{3}$) is weighed under water by the spring balance. The reading shown by the spring balance is -

(A) 15 N (B) 65 N (C) 150 N (D) 165 N

10. In the above question if the spring is cut, the acceleration of the person just after cutting the spring is

(A) zero (B) 1 m/s^2 (C) 9.8 m/s^2 (D) 0.91 m/s^2

Exercise-3

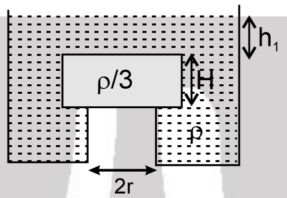
✍ **Marked Questions can be used as Revision Questions.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Comprehension-1

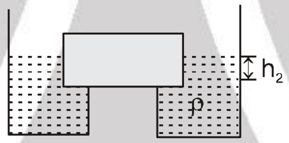
A wooden cylinder of diameter 4r, height H and density $\rho/3$ is kept on a hole of diameter 2r of a tank, filled with liquid of density ρ as shown in the figure.

1.✍ If level of the liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder the block starts moving up. At what value of h_1, will the block rise : **[IIT-JEE 2006, 5/184]**



(A) $\frac{4H}{9}$ (B) $\frac{5H}{9}$ (C) $\frac{5H}{3}$ (D) Remains same

2.✍ The block in the above question is maintained at the position by external means and the level of liquid is lowered. The height h_2 when this external force reduces to zero is **[IIT-JEE 2006, 5/184]**



(A) $\frac{4H}{9}$ (B) $\frac{5H}{9}$ (C) Remains same (D) $\frac{2H}{3}$

3.✍ If height h_2 of water level is further decreased then, **[IIT-JEE 2006, 5/184]**

(A) cylinder will not move up and remains at its original position.

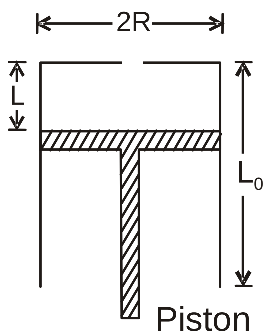
(B) for h_2 = H/3, cylinder again starts moving up

(C) for h_2 = H/4, cylinder again starts moving up

(D) for h_2 = H/5 cylinder again starts moving up

Comprehension-2

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0. **[IIT-JEE 2007, 4 × 3/184]**



4. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be

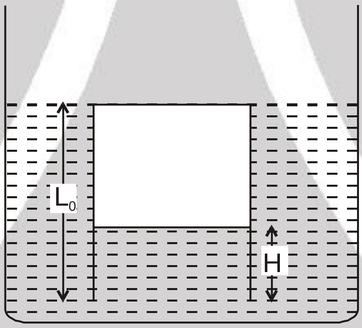
(A) P_0 (B) $\frac{P_0}{2}$ (C) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

5. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$

6. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(A) $\rho g (L_0 - H)^2 + P_0 (L_0 - H) + L_0P_0 = 0$ (B) $\rho g (L_0 - H)^2 - P_0 (L_0 - H) - L_0P_0 = 0$

(C) $\rho g (L_0 - H)^2 + P_0 (L_0 - H) - L_0P_0 = 0$ (D) $\rho g (L_0 - H)^2 - P_0 (L_0 - H) + L_0P_0 = 0$

7. **STATEMENT -1 :** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT -2 : In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **[IIT-JEE 2008, 3/162]**

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True.

8. **Column II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and and/or Y. Match these statements to the appropriate system(s) from **Column II.** **[IIT-JEE 2009, 8/160]**

Column I

(A) The force exerted by X on Y has a magnitude Mg.

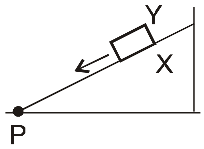
(B) The gravitational potential energy of X is continuously increasing,

(C) Mechanical energy of the system X + Y is continuously decreasing.

(D) The torque of the weight of Y about point P is zero.

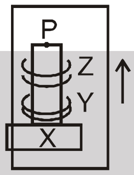
Column I

(p)



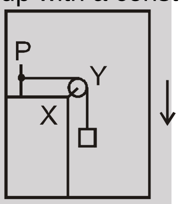
Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity.

(q)



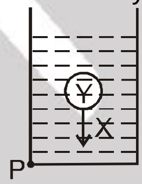
Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.

(r)



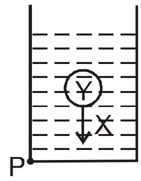
A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.

(s)



A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.

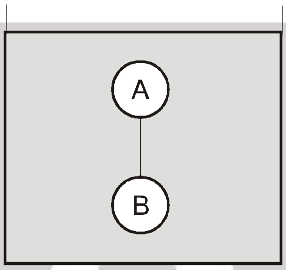
(t)



A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

9. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (**in mm**) of water level due to opening of the orifice. [Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension] **[IIT-JEE 2009, 4/160, –1]**

10*. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if **[IIT-JEE 2011, 4/160]**



(A) $d_A < d_F$ (B) $d_B > d_F$ (C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

11*. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **[JEE (Advanced)-2013, 3/60, –1]**

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$ (B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged. (D) the light sphere is completely submerged.

Paragraph for Question 12 to 13

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1mm respectively. The upper end of the container is open to the atmosphere.

12. If the piston is pushed at a speed of 5mms^{-1}, the air comes out of the nozzle with a speed of **[JEE (Advanced) 2014, 3/60, –1]**

(A) 0.1ms^{-1} (B) 1ms^{-1} (C) 2ms^{-1} (D) 8ms^{-1}

13. If the density of air is ρ_a and that of the liquid ρ_ℓ, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to **[JEE (Advanced) 2014, 3/60, –1]**

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a \rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) ρ_ℓ

14. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d = 1.2 m from the person. In the following, state of the lift's motion is given in List - I and the distance where the water jet hits the floor of the lift is given in List - II. Match the statements from List - I with those in List- II and select the correct answer using the code given below the lists. **[JEE (Advanced)-2014, 3/60, –1]**

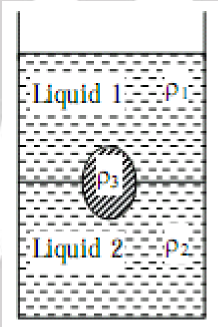
List -I	**List-II**
P. Lift is accelerating vertically up.	**1.** d = 1.2 m
Q. Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	**2.** d > 1.2m
R. List is moving vertically up with constant Speed	**3.** d < 1.2 m
S. Lift is falling freely.	**4.** No water leaks out of the jar

Code :

(A) P-2, Q-3, R-2, S-4 (B) P-2, Q-3, R-1, S-4 (C) P-1, Q-1, R-1, S-4 (D) P-2, Q-3, R-1, S-1

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

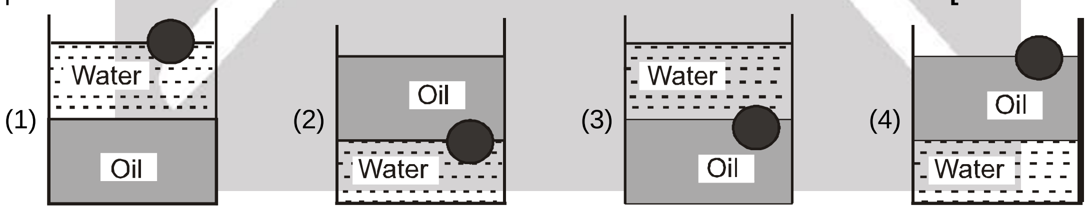
1. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. **[AIEEE 2008, 4/300]**



Which of the following is true for ρ_1, ρ_2 and ρ_3 ?

(1) $\rho_1 > \rho_3 > \rho_2$ (2) $\rho_1 < \rho_2 < \rho_3$ (3) $\rho_1 < \rho_3 < \rho_2$ (4) $\rho_3 < \rho_1 < \rho_2$

2. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position? **[AIEEE 2010, 4/144]]**



3. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to : **[AIEEE - 2011, 4/120, –1]**

(1) 5.0×10^{-3} m (2) 7.5×10^{-3} m (3) 9.6×10^{-3} m (4) 3.6×10^{-3} m

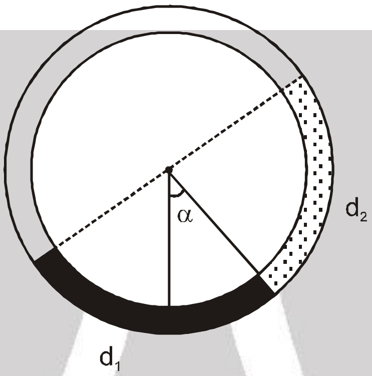
4. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) $2\pi\sqrt{\dfrac{\ell d}{\rho g}}$ (2) $2\pi\sqrt{\dfrac{\ell \rho}{dg}}$ (3) $2\pi\sqrt{\dfrac{\ell d}{(\rho - d)g}}$ (4) $2\pi\sqrt{\dfrac{\ell \rho}{(\rho - d)g}}$

5. A uniform cylinder of length L and mass M having cross - sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is :**[JEE (Main) 2013, 4/120,–1]**

(1) $\frac{Mg}{k}$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (3) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (4) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

6.✍ There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is : **[JEE(Main) 2014, 4/120, –1]**



(1) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (2) $\frac{1+\cos\alpha}{1-\cos\alpha}$ (3) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (4) $\frac{1+\sin\alpha}{1-\cos\alpha}$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. Sharp kinife applies more pressure as compare to blunt knife because of lesser area of contact.

A-2. It has high specific gravity.

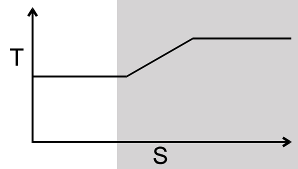
A-3. 500 kg/m^3, 0.5

A-4. If g = 10m/s^2, 253200 N/m^2

Section (B) :

B-1. 10 cm **B-2.** 19.6 m, 4 sec

B-3.



Section (C) :

C-1. 6.43×10^{-4} m^3/s

C-2. $v = \frac{10}{\sqrt{6}}$ m/s = 4.1 m/s; $v' = \frac{50}{\sqrt{6}}$ m/s = 21 m/s ;

Av = 8.1 x 10^{-3} m^3 /sec

C-3 (i) 25 cm/s, (ii) 50 cm/s (iii) 93.75 N/m^2

C-4. (i) 25 cm/s, (ii) 50 cm/s (iii) zero

C-5. 187.5 N/m^2

C-6. $v_{max} = \left(\frac{2p_{atm}}{\rho}\right)^{1/2}$

PART - II

Section (A) :

A-1.	(C)	**A-2.**	(A)	**A-3.**	(A)
A-4.	(B)	**A-5.**	(i) (A), (ii) (C)		

Section (B) :

B-1.	(D)	**B-2.**	(A)	**B-3.**	(A)
B-4.	(A)	**B-5.**	(C)	**B-6.**	(C)
B-7.	(A)	**B-8**	(C)		

Section (C)

C-1.	(C)	**C-2.**	(C)	**C-3.**	(C)
C-4.	(A)	**C-5.**	(B)	**C-6.**	(A)
C-7.	(B)	**C-8.**	(D)		

PART - III

1. A → p ; B → q ; C → t ; D → s

2. A → q ; B → p ; C → r ; D → s

EXERCISE-2

PART - I

1.	(D)	**2.**	(B)	**3.**	(B)
4.	(A)	**5.**	(B)	**6.**	(A)
7.	(C)	**8.**	(A)	**9.**	(B)
10.	(B)	**11.**	(A)	**12.**	(B)
13.	(A)	**14.**	(B)	**15.**	(D)
16.	(B)	**17.**	(A)	**18.**	(A)
19.	(D)	**20.**	(C)	**21.**	(B)

PART - II

1.	1	**2.**	10	**3.**	30
4.	5	**5.**	2	**6.**	2
7.	2	**8.**	25	**9.**	32

PART - III

1.	(BC)	**2**	(AC)	**3.**	(ACD)
4.	(CD)	**5.**	(ABD)	**6.**	(D)
7.	(BC)	**8.**	(AB)		

PART - IV

1.	(A)	**2.**	(D)	**3.**	(B)
4.	(A)	**5.**	(B)	**6.**	(B)
7.	(B)	**8.**	(A)	**9.**	(C)
10.	(D)				

EXERCISE-3

PART - I

1.	(C)	**2.**	(A)	**3.**	(A)
4.	(A)	**5.**	(D)	**6.**	(C)
7.	(A)				

8. (A) → (p), (t) ; (B) → (q), (s), (t); (C) → (p), (r), (t); (D) → (q)

9.	6	**10.**	(ABD)	**11.**	(AD)
12.	(C)	**13.**	(A)	**14.**	(C)

PART - II

1.	(3)	**2.**	(2)	**3.**	(4)
4.	(1)	**5.**	(3)	**6.**	(3)

High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A ball of density d is dropped onto a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L. **[JEE-1992, 8 Marks]**

(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.

(b) Is the motion of the ball simple harmonic?

(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid ?
Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

2. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ as shown in figure. The height of the liquid in one vessel is h_2 and other vessels h_1, the area of either base is A. Find the work done by gravity in equalizing the levels when the two vessels are connected.

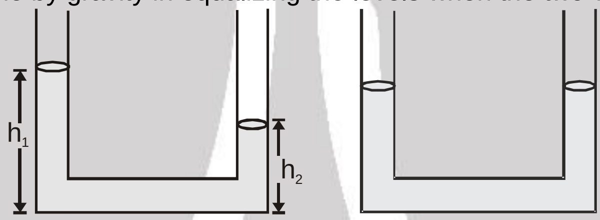
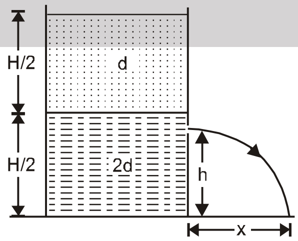


Figure (1) Figure (2)

3. A cyllindrical wooden stick of length L, and radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ (>ρ). **[JEE - 1999, 10/100]**

4. A container of large uniform cross–sectional area A resting on a horizontal surface, holds two immiscible, non–viscous and incompressible liquids of densities d and 2d, each of height $\frac{H}{2}$ as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0. **[JEE - 1995, 5 + 5M]**

(a) A homogeneous solid cylinder of length $L\left(L < \frac{H}{2}\right)$ cross–sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid–liquid interface with the length $\frac{L}{4}$ in the denser liquid. Determine:



(i) The density D of the solid and (ii) The total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s (s << A) is punched on the vertical side of the container at a height $h\left(h < \left(\frac{H}{2}\right)\right)$. Determine :

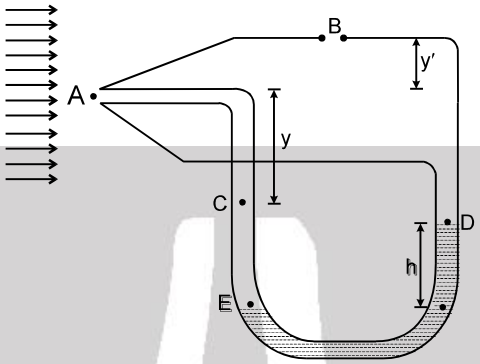
(i) The initial speed of efflux of the liquid at the hole

(ii) The horizontal distance x travelled by the liquid initially and

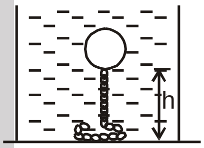
(iii) The height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m. [Neglect air resistance in these calculations]

5. A container of cross-section area 'S' and height 'h' is filled with mercury up to the brim. Then the container is sealed airtight and a hole of small cross section area ' S/n ' (where 'n' is a positive constant) is punched in its bottom. Find out the time interval upto which the mercury will come out from the bottom hole. [Take the atmospheric pressure to be equal to h_0 height of mercury column: $h > h_0$]

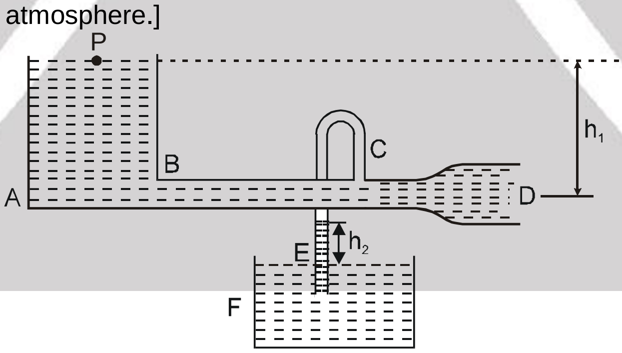
6. A Pitot tube is shown in figure. Wind blows in the direction shown. Air at inlet A is brought to rest, whereas its speed just outside of opening B is unchanged. The U tube contains mercury of density ρ_m. Find the speed of wind with respect to Pitot tube. Neglect the height difference between A and B and take the density of air as ρ_a.



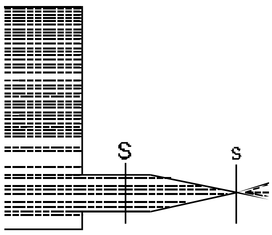
7. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density 1/3 while the other end is free. The sphere along with the chain is immersed in a deep lake. If specific density of iron is 7, the height h above the bed of the lake at which the sphere will float in equilibrium is (Assume that the part of the chain lying on the bottom of the lake exerts negligible force on the upper part of the chain.) :



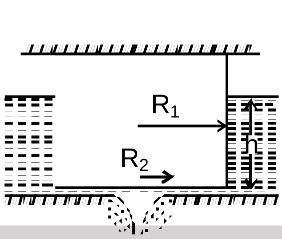
8. Two very large open tanks A & F both contain the same liquid. A horizontal pipe BCD, having a small constriction at C, leads out of the bottom of tank A, and a vertical pipe E containing air opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section area at C is one-half that at D, and if D is at distance h_1 below the level of the liquid in A, to what height h_2 will liquid rise in pipe E? Express your answer in terms of h_1. [Neglect changes in atmospheric pressure with elevation. In the containers there is atmosphere above the water surface and D is also open to atmosphere.]



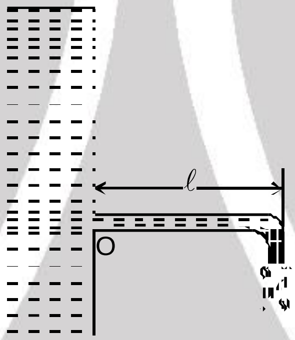
9. A side wall of a wide open tank is provided with a narrowing tube (as shown in figure) throught which water flows out. The cross-sectional area of the tube decrease from S = 3.0 cm^2 to s = 1.0 cm^2. The water level in the tank is h = 4.6 m higher than that in the tube. Neglecting the viscosity of the water, find the horizontal component of the force tending to pull the tube out of the tank.



10. The horizontal bottom of a wide vessel with an ideal fluid has a round orifice of radius R_1 over which a round closed cylinder is mounted, whose radius $R_2 > R_1$. The clearance between the cylinder and the bottom of the vessel is very small, the fluid density is ρ. Find the static pressure of the fluid in the clearance as a function of the distance r from the axis of the orifice (and the cylinder), if the height of the fluid is equal to h.



11. Water flows out of a big tank along a tube bent at right angles, the inside radius of the tube is equal to r = 0.50 cm. The length of the horizontal section of the tube is equal to ℓ = 22 cm.The water flow rate is Q = 0.50 litres per second. Find the moment of reaction forces of flowing water, acting on the tube's walls, relative to the point O.



HLP Answers

1. (a) $\frac{t_1 d_L}{d_L - d}$ (b) No (c) $v = g\frac{t_1}{2}$ = constant

2. $\frac{gA\rho}{4}(h_1 - h_2)^2$ **3.** $m \geq \pi r^2 L\,(\sqrt{\rho\sigma} - \rho)$

4. (a) (i) Density = $\frac{5}{4}d$

(ii) Pressure = $P_0 + \frac{1}{4}(6H + L)\,dg$

(b) (i) $v = \sqrt{\frac{g}{2}(3H - 4h)}$ (ii) $x = \sqrt{h(3H - 4h)}$

(iii) $x_{max} = \frac{3}{4}H$, $h_{max} = \frac{3H}{8}$

5. $t = n\sqrt{\frac{2}{g}(h - h_0)}$ **6.** $v = \sqrt{\frac{2(\rho_m - \rho_a)gh}{\rho_a}}$

7. $\frac{7m}{3\lambda}$ **8.** $h_2 = 3h_1$

9. $F = \rho gh\,(S - s)^2/S = 6N$

10. $p = p_0 + \rho gh\,(1 - R_1^2/r^2)$, where $R_1 < r < R_2$, p_0 is the atmospheric pressure.

11. $N = \rho\ell Q^2/\pi r^2 = 0.7$ N.m.

HINT & SOLUTION OF FLUID MECHANICS
EXERCISE-1
PART - I

A-1. Sharp kinife applies more pressure as compare to blunt knife because of lesser area of contact.

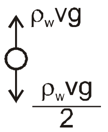
A-2. It has high specific gravity.

A-3. $p = h\rho g$
$15 \times 10^3 = 3 \times \rho \times 10$
$\rho = 500 \text{ kg/m}^3$
$s = \frac{\rho}{\rho_w} = \frac{500}{1000} = 0.5$

A-4. $P_x + \rho_\omega g\left(\frac{175}{100}\right) - \rho_{Hg} g\left(\frac{112}{100}\right) + \rho_\omega g\left(\frac{75}{100}\right) - \rho_{Hg} g\left(\frac{88}{100}\right) - \rho_\omega g\left(\frac{62}{100}\right) = P_y$
Solving $P_x - P_y = 253200 \text{ N/m}^2$ ($g = 10 \text{ m/sec}^2$)

A-5. Initially $\left[\frac{0+\rho ga}{2}\right](a)^2 + \left[\frac{\rho ga + 3\rho ga}{2}\right](a^2) = 2.5\ \rho ga^3$
Finally $\left[\frac{0+\rho ga}{2}\right](2a^2) = 2\rho ga^3$
Difference = $0.5\ \rho ga^3$ [Which is one fifth of initial]

B-1. $\rho(a^2)(2) = 200$ g
but $\rho = 1 \text{ gm/cm}^3$
$a = 10$ cm.

B-2. In water $a = g$


So, by symmetry in air and water in acceleration distance travelled = 19.6 m.
$t = 2\sqrt{2h/g} = 2\sqrt{2 \times 19.6/9.8} = 4$ sec.

B-3. Till the top edge reaches the surface of water. Tension = weight – buoyant force and buoyant force till then is constant. When plate is partially in water and partially in air, then as plate move up buoyant force decreases and when plate comes out tension again becomes constant or T = Weight.

C-1. $A_1v_1 = A_2v_2$ ....(i)
$P_1 + \frac{1}{2}\rho v_1^2 + 0 = P_2 + \frac{1}{2}\rho v_2^2 + 0$ ....(ii)
Given, $P_1 - P_2 = 10 \text{ N/m}^2$
$\rho = 1.25 \times 10^3$
$r_1 = 0.1$ m
$r_2 = 0.04$ m
Solving, rate of flow of glycerin = $A_1V_1 = A_2V_2 = 6.43 \times 10^{-4} \text{ m}^3\text{/sec}$.

C-2. $av' = Av$
$A = 5a$
$v' = 5v$
$P_A + \frac{1}{2}\rho v_A^2 + 0 = P_a + \frac{1}{2}\rho v_a^2$
but $P_a = 0$ and $P_A = 2$ atm
Solving $Av \equiv 8.1 \times 10^{-3}$ m^3/sec.

C-3. $A_1v_1 = A_2v_2$
(i) 10 cm^3/s $= 40$ mm$^2 \times v_x$ or $v_x = 25$ cm/s
(ii) 10 cm^3/s $= 20$ mm$^2 \times v_y$ or $v_y = 50$ cm/s
$P_x + \frac{1}{2}\rho v_x^2 = P_y + \frac{1}{2}\rho v_y^2$
$P_x - P_y = \frac{1}{2}\rho v_y^2 - \frac{1}{2}\rho v_x^2 = \frac{1}{2} \times 1000 \left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right]$
or $P_x - P_y = 93.75$ N/m^2 .

C-4. $A_1v_1 = A_2v_2$
$P_y + \frac{1}{2}\rho v_y^2 + 0 = P_x + \frac{1}{2}\rho v_y^2 + \rho gh.$
$P_x - P_y = \frac{1}{2}\rho(v_y^2 - v_x^2) - \rho gh$
$P_x - P_y = \frac{1}{2}1000\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right] - 1000 \times 10 \times \frac{15}{1600}$
$P_x - P_y = 0$

C-5. $A_1v_1 = A_2v_2$
$P_y + \frac{1}{2}\rho v_y^2 + \rho gh = P_x + \frac{1}{2}\rho v_x^2 + 0.$
$P_x - P_y = \frac{1}{2}\rho(v_y^2 - v_x^2) + \rho gh$
$P_x - P_y = \frac{1}{2}1000\left[\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2\right] + 1000 \times 10 \times \frac{15}{1600}$
$P_x - P_y = 187.5$ N/m^2

C-6. through Bernoulli's equation
If pressure becomes zero, then velocity will be maximum
$P_{atm} + 0 = 0 + \frac{1}{2}\rho v_{max}^2 \quad v_{max} = \left(\frac{2p_{atm}}{\rho}\right)^{1/2}$

PART - II

A-1. $\rho g(H - h)$
Thrust is equal to pressure at that point.

A-2. $F = [\rho gh]\,[A]$
$= (1000)\,(10)\,(6)\,(10)\,(8).$

A-3. $\frac{m_1 g}{A_1} = \frac{m_2 g}{A_2}$

Solving $m_2 = 3.75$ kg.

A-4. $W_A > W_B$ as mass of water in A is more than in B

$P_A = P_B$

Area of A = Area of B

or P_A $Area_A = P_B$ $Area_B$

or $F_A = F_B$.

A-5. (i) $a = a_0 (\hat{i} - \hat{j} + \hat{k})$

As there is no gravity; the pressure difference will be only due to the acceleration.
At point B the pseudo force is maximum hence pressure is maximum.
(ii) At point H the pseudo force is minimum hence pressure is minimum

B-1. $\Delta v = v_f - v_i = \frac{m}{y} - \frac{m}{x}$

B-2. $mg = 60$ ....(i)

$mg - \rho_l vg = 40$ ....(ii)

$\frac{mg - \rho_\ell vg}{mg} = \frac{2}{3}$ or $\frac{\rho_0}{\rho_\ell} = 3$

where ρ_0 = density of the block and ρ_ℓ = density of the liquid.

B-3. $\Delta U = mgh$

$\Delta U = \sigma_b Vgh$

B-4. $W_{app} = mg - F_B$

$W_{app} = \rho Vg - \rho_w Vg$

$= (\rho - \rho_w)\ Vg$

$= (7\rho_w - \rho_w)\ Vg = 6\ \rho_w\ Vg$

B-5. $10^3 \times \frac{4}{5} + 13.5 \times 10^3 \times \frac{1}{5} = \rho \times 1$

or $\rho = 3.5 \times 10^3$ kg/m^3

B-6. $[36 - \rho_\ell v_l]g = [48 - \rho_\ell v_2]g$

$$\left[36 - \rho_\ell\left(\frac{36}{9}\right)\right]g = \left[48 - \rho_\ell\left(\frac{48}{\rho_0}\right)\right]g$$

Solving

$\rho_0 = 3$.

B-7. In stable equilibrium the object comes to its original state if disturbed.

B-8. As, weight = Buoyant force

$mg = [100 \times 6 \times 0.6\ g] + (100 \times 1 \times 4)g$ $\therefore$ $m = 760$ gm.

C-1. $R = vt$

$= \sqrt{2gD}\ \sqrt{\frac{2(H-D)}{g}}$

$= 2\sqrt{D(H-D)}$.

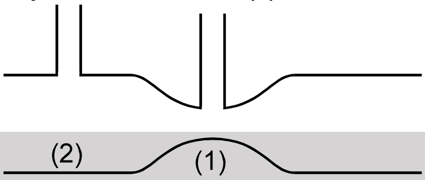
C-2. $x = 2\sqrt{h(H-h)}$

for x_{max} $\frac{dx}{dh} = 0$ or $h = \frac{H}{2}$

C-3. from equation of continuity

$(A \times 3) = (A \times 1.5) + (1.5\ A \times V) \Rightarrow V = 1\ m/s^2$

C-4. From continuity equation, velocity at cross-section (1) is more than that at cross-section (2).



Hence; $P_1 < P_2$

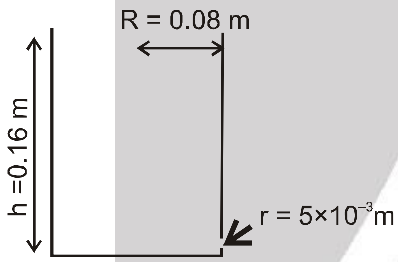
C-5. $F_{thrust} = \rho a v^2$

$F_{net} = F_1 - F_2 = a\rho[2g(h_1 - h_2)]$

$= a\rho(2gh)$

or $F \propto h$

C-6.



$A_1 v_1 = A_2 v_2$

$\pi R^2\ dh/dt = \pi r^2\ v$ ....(i)

$v = \sqrt{2gh}$ ....(ii)

from equation (ii) put the value of v in equation (i)

$\pi R^2\ dh/dt = \pi r^2\ \sqrt{2gh}$

$$\Rightarrow \int \frac{R^2 dh}{r^2\sqrt{2gh}} = \int dt$$

$$\frac{R^2}{r^2\sqrt{2g}} \int_h^0 \frac{dh}{\sqrt{h}} = \int_0^T dt$$

$$T = \frac{R^2}{r^2}\sqrt{\frac{2h}{g}}$$

on solving t = 46.26 second.

C-7. Thrust on curved surface $= \langle p \rangle 2\pi RH$

$= 15\ .\ 2\pi R(10)$

Force on bottom

$= \langle p \rangle \pi R^2 = 20\ .\ \pi R^2$

$15\ 2\pi R(10) = 20\ \pi R^2$

$\therefore R = 15\,m$

C-8. Thrust force

$F = \rho A v_{rel}^2$

$v_{rel} = (v - V)$

$\therefore\ F \alpha\ v_{rel}^2$

PART - III

1. Pressure varies with height $\Rightarrow P = \rho gh$

and is horizontal with acceleration $\Rightarrow P = \rho \ell a$

so on (A) ρgh part is zero while average force of ρax is

$\left[\frac{0+\rho\ell a}{2}\right] [\ell^2]$

$= \frac{\ell\rho a}{2}(\ell^2) = \frac{(\rho\ell^3)}{2}\ a = \frac{ma}{2}$

In (B) $\rho\ell a$ part is zero while average force of ρgx is

$\left[\frac{0+\rho g\ell}{2}\right] \left[\ell^2\right] = \frac{\rho g}{2}\ (\ell^3)$

$= \frac{\rho(\ell^3)}{2}(g) = \frac{mg}{2}$

Similarly for other part

2. (A) On ABCD avg pressure $= \left[\frac{0+\rho_1 gh}{2}\right]$

(A) ABCD $= \left[\frac{0+\rho_1 gh}{2}\right]$ So $F = \left[\frac{\rho_1 gh}{2}\right]$ $[\ell h]$ $= \frac{\rho_1 gh^2\ell}{2}$

(B) No contact of ρ_2 and not any pressure on ABCD due to ρ_2

(C) On CDEF due to ρ_1, at every point pressure is $\rho_1 gh$ so average is also $\rho_1 gh$

so $F = (\rho_1 gh)(h\ell) = \rho_1 gh^2\ell$

(D) On CDEF force due to liquid of density ρ_2 is $\frac{\left[\rho_2 gh^2\ell\right]}{2}$

EXERCISE - 2

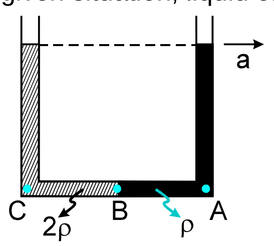
PART - I

1. $F_{th} = \frac{\sqrt{2}dp}{dt} = \sqrt{2}V\frac{dm}{dt} = \sqrt{2}V\ [\rho\ L\ ] = \sqrt{2}\rho VL$

2. Pressure exerted by fluid at closed end B is $P = \rho g\ell$

$\therefore$ Force exerted by fluid at closed end B is $F = PA = \ell\rho g\ A_0$

3. For the given situation, liquid of density 2 ρ should be behind that of ρ.



From right limb :

$P_A = P_{atm} + \rho\, gh$

$P_B = P_A + \rho a \frac{\ell}{2} = P_{atm} + \rho gh + \rho a \frac{\ell}{2}$

$P_C = P_B + (2\rho) a \frac{\ell}{2} = P_{atm} + \rho gh + \frac{3}{2} \rho a \ell$ .... (1)

But from left limb :

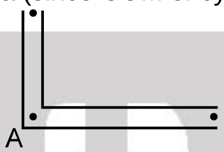
$P_C = P_{atm} + (2\rho) gh$ .... (2)

From (1) and (2) :

$P_{atm} + \frac{3}{2}\rho gh + \rho a\ell = P_{atm} + 2\rho gh \Rightarrow h = \frac{3a}{2g}\ell$ **Ans.**

4. No sliding $\Rightarrow$ pure rolling

Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

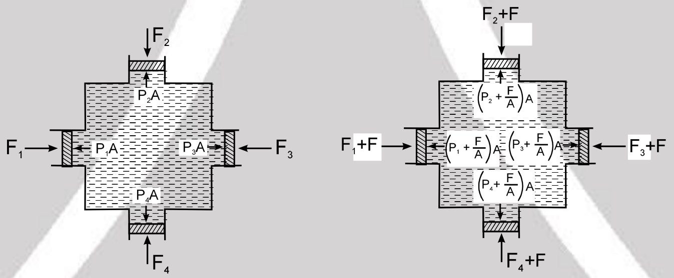


$P_A = P_{atm} + \rho (2a) L$ (From horizontal limb)

Also ; $P_A = P_{atm} + \rho g H$ (From vertical limb) $\Rightarrow a = \frac{gH}{2L}$ **Ans.**

5. (B) As long as $\rho \le \rho_W$, pressure at the bottom of the pan would be same everywhere, according to the Pascal's law.

6. The four piston are initially in equilibrium. As additional force F is applied to each piston, the pressure in fluid at each point must be increased by $\frac{F}{A}$ so that each piston retains state of equilibrium.



Thus the increment in pressure at each point is $\Delta P = F/A$ (by Pascal's law)

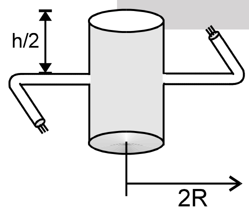
7. Since not touching,

So, $R = F_b = \rho_l(vg) = 40g$.

$R' - R = 80g - 40g = 40g$

Hence R′ will be 40g more than R

8.



Velocity of efflux of water (v) = $\sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

Force on ejected water = Rate of change of momentum of ejected water.

$= \rho (av) (v) = \rho av^2$

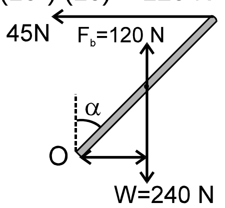
Torque of these forces about central line

$= (\rho av^2)\, 2R.2 = 4\rho av^2 R = 4\rho\, agh R$

9. F.B.D. of rod

$W = (0.012)\,(1)\,(2 \times 10^3)\,(10) = 240$ N

$F_b = (0.012)\,(1)\,(10^3)\,(10) = 120$ N



Torque about O

(For equilibrium

$(240 - 120)\left(\frac{\sin\alpha}{2}\right) = 45\,(\cos\alpha)$

$\Rightarrow \tan\alpha = \frac{90}{120} = \frac{3}{4} \Rightarrow \alpha = 37°$

10. Torque about CM

$F_b.\frac{\ell}{4} = I\,\alpha$

$\Rightarrow \quad \alpha = \frac{1}{I}(\pi r^2)\,(\ell)\,(\rho)\,(g).\,\frac{\ell}{4} \qquad \alpha = \frac{\pi r^2 \ell^2 g\rho}{4I}$

'α' will be same for all points on cylinder

Hence (B).

11. Increasing the temperature of water from 2^0C to 3^0C increases its density while decreases the density of iron.

Hence the bouyant force increases.

12. $y = \frac{\omega^2 r^2}{2g}$

Put values and get y = 2cm.

13. Buoyant force = $F_b = V_{sub}\,.\,\rho_\ell\,.\,g$

where, V_{sub}, ρ_ℓ and g all are same w.r.t. O_1 and O_2.

Hence (A)

14. $A_1V_1 = A_2V_2$ or $A \times V_1 = 2A\sqrt{\frac{g\ell}{2}}$ or $V_1 = \sqrt{2g\ell}$

$\frac{1}{2}\rho\left[V_1^2 - V_2^2\right] = \rho g\ell \sin\theta$

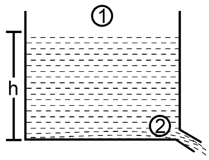
$\Rightarrow \quad \frac{1}{2}\left[2g\ell - \frac{g\ell}{2}\right] = g\ell \sin\theta$

on solving $\sin\theta\,\frac{3}{4}$.

15. The velocity of fluid at the hole is $V_2 = \sqrt{\dfrac{2gh}{1+(a^2/A^2)}}$

Using continuity equation at the two cross-sections (1) and (2) :

$V_1 A = V_2 a \quad \Rightarrow \quad V_1 = \dfrac{a}{A} V_2$



$\Rightarrow$ Acceleration (of top surface) $= -V_1 \dfrac{dV_1}{dh}$

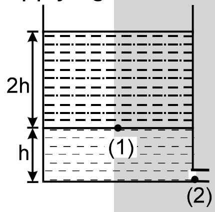
$= -\dfrac{a}{A} V_2 \dfrac{d}{dh}\left(\dfrac{a}{A} V_2\right)$

$a_1 = -\dfrac{a^2}{A^2} V_2 \dfrac{dV_2}{dh} = -\dfrac{a^2}{A^2}\sqrt{2gh}\sqrt{2g}.\dfrac{1}{2\sqrt{h}} \Rightarrow a_1 = \dfrac{-ga^2}{A^2}$

16. Pressure at (1) :

$P_1 = P_{atm} + \rho\, g\, (2h)$

Applying Bernoulli's theorum between points (1) and (2)



$[P_{atm} + 2\,\rho\, g\, h] + \rho g(2h) + 1/2\,(2\,\rho)\,(0)^2$

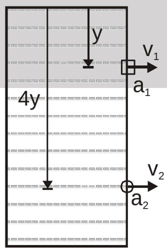
$= P_{atm} + (2\,\rho)\, g\,(0) + 1/2\,(2\,\rho)\, v^2 \Rightarrow \quad v = 2\sqrt{gh}$ **Ans.**

17. by $A_1 V_1 = A_2 V_2$

$\left(\dfrac{\pi D_1^2}{4}\right) V_1 = \left(\dfrac{\pi D_2^2}{4}\right) V_2; \quad V_1 = 4V_2$

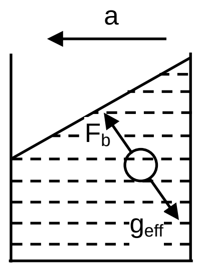
18. Velocity of efflux at a depth h is given by $V = \sqrt{2gh}$

Volume of water following out per second from both the holes are equal



$\therefore \quad a_1V_1 = a_2V_2$

or $\quad (L^2)\sqrt{2g(y)} = \pi R^2\sqrt{2g(4y)} \quad$ or $\quad R = \dfrac{L}{\sqrt{2\pi}}$

19. 

w.r.t. container

20. $\frac{\rho A}{\rho \ell} = \frac{1}{2}$ $\Rightarrow$ $\frac{\rho A}{\rho B} = \frac{1}{3}$

$\rho_A Vg + \rho_B Vg = \rho \ell_2 Vg$

$\rho_A + \rho_B = 2\rho_e$

$\rho_B = 3\rho_A$

$\rho_\ell = 2\rho_A$

$\rho_\ell : \rho_A : \rho_B = 2 : 1 : 3$

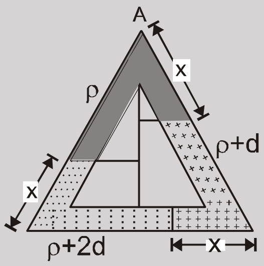
21. $\frac{4}{3}\pi\left(1000r^3 - 729r^3\right)\rho = \frac{2}{3}\pi 1000r^3 0.8\rho\omega$

$271\rho = 4\omega\rho_\omega$

$\rho = 1.48\rho_\omega = 1.48 \text{ gm/cm}^3$

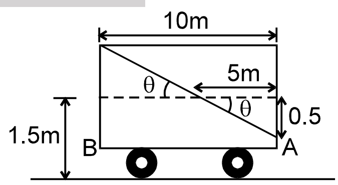
PART - II

1. $(\rho + 2d)\ x \sin 60 + \rho(\ell - x)\ \sin 60^o = (\rho + d)\ (\ell - x) \sin 60^o + \rho x \sin 60^o$
On solving



$x = \frac{\ell}{3} = \frac{3}{3} = 1$

2. $\tan\theta = \frac{0.5}{5} = \frac{1}{10}$



$\frac{a}{g} = \frac{1}{10}$

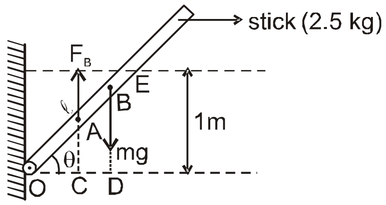
$a = 1 \text{ m/s}^2$

$v = u + at$

$10 = 0 + 1 \times t$

$t = 10$ second.

3. OE = ℓ



$\ell = \frac{1}{\sin\theta}$ ...(i)

OD = 2 cos θ ...(ii)

OC = $\frac{\ell}{2}\cos\theta = \frac{\cos\theta}{2\sin\theta}$ ...(iii)

$F_B = \rho_w A\ell g$

torque about O

mg (OD) = $\rho_w A\ell g$(OC)

on solving

$\sin^2\theta = \frac{1}{4}$

θ = 30 º **Ans.**

For height $\sin^2\theta < \frac{h^2}{4}$ (h is depth of water)

For θ = 90º, h > 2

4. Breaking force = Breaking stress × area of wire

= $7 \times 10^6 \times 10^{-6}$ = 7N

weight of block = vol × density × g

= $10^{-3} \times (1.5 \times 10^3) \times 10$ = 15 N.

The upthrust when the liquid level has descended by x cm.

= $100 (10 - x) \cdot 10^{-6} \cdot 10^3 \cdot 10$

= 10 – x Newton.

∴ Net downward force on the block = 15 – (10 – x) = 5 + x = Tension T

∴ The wire will break when 5 + x = 7.

i.e. when x = 2cm

Let the level descends by 2 cm in t time

then 2t = (200 – 100).2

t = 100 sec. ......Ans.

20x = 100 ∴ x = 5

5. $\rho a V^2 = \mu N$

1000 × a (2g ×2) = 0.4 × 100 × 10

$1000 \times \pi \frac{d^2}{4} \times 4g = 400$ or $d = \frac{0.2}{\sqrt{\pi}}$

6. vt = 80 m ...(i)

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{10}}$ = 2 second ....(ii)

from (i) and (ii) equ.

v = 40 m/s. ....(iii)

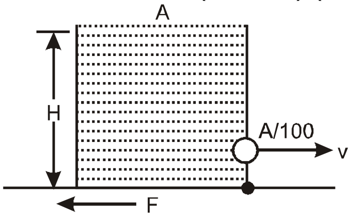
$P_0 + \rho g h = P_{atm} + \frac{1}{2}\rho v^2$ ...(iv)

on solving (iv) equation

$P_0 = 2.78 \times 10^5$ Nm^{-2} = $(1.39)\alpha \times 10^5$

α = 2 **Ans.**

7. Mass of water = (Volume) (density)



$\therefore$ $m_0 = (AH)\ r$

$\therefore$ $H = \dfrac{m_0}{A\rho}$

Velocity of efflux, $V = \sqrt{2gH} = \sqrt{2g\dfrac{m_0}{A\rho}} = \sqrt{\dfrac{2m_0 g}{A\rho}}$

Thrust force on the container due to draining out of liquid from the bottom is given by,
F = (density of liquid) (area of hole) (velocity of effux)^2
$(F = \rho a V^2)$

$F = \rho(A/100)V^2 = \rho(A/100)\left(\dfrac{2m_0 g}{A\rho}\right)$

$F = \dfrac{m_0 g}{50}$

$\therefore$ Acceleration of the container, $a = F/m_0 = g/50 = \dfrac{1}{5} = 0.2\ m/s^2$

So $x = 2$ **Ans.**

8. Given : $A_1 = 4 \times 10^{-3}\ m^2$, $A_2 = 8 \times 10^{-3}\ m^2$
$h_1 = 2m$, $h_2 = 5m$
$v_1 = 1$ m/s and $\rho = 10^3\ kg/m^3$
From continuity equation, we have

$A_1 v_1 = A_2 v_2$ or $v_2 = \left(\dfrac{A_1}{A_2}\right) v_1$

or $v_2 = \left(\dfrac{4\times10^{-3}}{8\times10^{-3}}\right)$ (1 m/s)

$v_2 = \dfrac{1}{2}$ m/s

Applying Beronulli`s equation at section 1 and 2

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or $P_1 - P_2 = \rho g\ (h_2 - h_1) + \dfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$ ...(2)

(i) Work done per unit volume by the pressure as the fluid flows from P to Q.

$W_1 = P_1 - P_2$

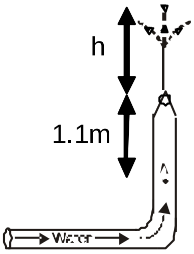
$= \rho g\ (h_2 - h_1) + \dfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$ [From Eq. (1)]

$= \left\{(10^3)(9.8)(5-2) + \dfrac{1}{2}(10^3)\left(\dfrac{1}{4} - 1\right)\right\}$ J/m^3 = [29400 – 375] J/m^3 = 29025 J/m^3

$1161\ \alpha = 29025$ J/m^3

$\alpha = 25$

9.



$A_1v_1 = A_2v_2$

$Av_A = \frac{A}{4}v_n$

$v_n = 4v_A$ ....(i)

$P_A + \frac{1}{2}\rho v_A^2 = \frac{1}{2}\rho v_n^2 + \rho g\hbar$ ...(ii)

$v_n^2 / 2g = \hbar$ ....(iii)

On solving (i), (ii) & (iii) equation we get $\hbar = 3.2$ m **Ans.**

PART - III

1. PV = constant
(Assumed isothermal process)

2. In a static fluid, pressure remains same at the same level, ie, pressure do not vary with x-coordinate. Hence (C).

3. $P = \rho(2h)g$

$\frac{F}{A_2} = \rho\,(2h)\,g$

$F_{base} = 2h\,\rho\,g\,A_2$

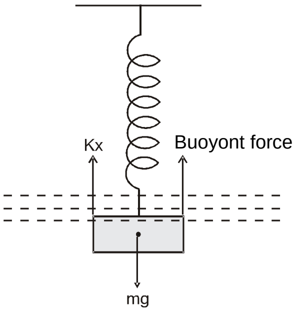
$F_{wall} = h\,\rho\,g\,[A_2 - A_1]$, at the level x

4. Assuming that the block is completely submerged in water, then
$F_b = 1000 > mg(920)$ So, not possible
Let complete in oil
$F_b = (0.6)\,(4)\,(1000 + (1)\,(6)\,(100) = 840$
$F_b < mg$ So, not possible
So, let 'x' part in oil and remaining in water
$920 = [(1)\,(10 - x) + (0.6)\,(x)]\,100$
$9.2 = 10 - x + 0.6x$
$0.4x = 0.8$
x = 2 cm.

5. $F_b = v\rho_{liq}g$
'g' is different on moon and on the earth.
Hence only (iii) is a correct statement.
Hence (D).

6. ℓ will decrease because the block moves up. h will decrease because the coin will displace the volume of water (V_1) equal to its own volume when it is in the water whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the density of water $V_1 < V_2$.

7. $Kx = 10 \times 10 - \frac{1000}{2000} \times 10 \times 10$



Kx = 50 N

$U_{stored} = \frac{1}{2} \times (100) \left(\frac{50}{100}\right)^2 = \frac{1}{2} \times \frac{2500}{100} = 12.5$ J

8. $x = 2\sqrt{H(H-h)}$

$x_1 = 2\sqrt{70 \times 20}$

$x_2 = 2\sqrt{60 \times 30}$

$x_3 = 2\sqrt{40 \times 50}$

$x_4 = 2\sqrt{50 \times 40}$ or $x_3 = x_4$ = maximum

PART - IV

1. As, $dm = A\rho_W v\,dt$

$\Rightarrow \frac{dm}{dt} = A\rho_W v$

$\Rightarrow \frac{dm}{dt} = V\rho_W \pi \frac{D^2}{4}$

where 'D' is the diameter of stream.

2. $V_1A_1 = V_2A_2$

$\frac{\pi v_0 D_0^2}{4} = \frac{\pi v D^2}{4} \Rightarrow D = D_0\sqrt{\frac{v_0}{v}}$.

3. $v = \sqrt{u^2 + 2gh} = \sqrt{2g(b+x)}$.

4. Applying continuity equation at points with diameter D_0 & D :

$= \sqrt{2gb}.\left[\frac{\pi.D_0^2}{4}\right] = \sqrt{2g(b+x)}\left[\frac{\pi D^2}{4}\right]$

$\Rightarrow D = D_0\left[\frac{b}{b+x}\right]^{1/4}$

5. Solving the preceding formula for the tank height h gives :
$h = x(D/D_0)^4/(1 - (D/D_0)^4) = x\,D^4 / (D_0^4 - D^4)$
Substituting the given parameter values gives
$h = (0.3)(0.009^4) / (0.01^4 - 0.009^4) = 0.57$ m
So the height of the water above the tap is 0.57 m or 57 cm.

6. Apparent weight ($W_{app.}$) = $W - V\rho_\ell g$
Since, $W_{app.\,(Ram)} > W_{app.\,(Shyam)}$
$\Rightarrow W_{(Ram)} > W_{(Shyam)}$
Therefore, from given passage shyam has more fat than Ram.

7. $V_1 > V_2 \Rightarrow W_{app.\,(1)} < W_{app.\,(2)}$
(Since $W_{app.} = W - V\rho_\ell g$)
Hence (B)

8. $\rho_{Salt\ water} > \rho_{Fresh\ water} \Rightarrow W_{app.\,(s)} < W_{app.\,(F)}$
Hence (A)

9. Let 'V' be the total volume of the person
Then ; $\left(\frac{V}{4}\right)(0.4 \times 10^3) + \left(\frac{3}{4}V\right)\left(\frac{4}{3}\times 10^3\right) = 165$

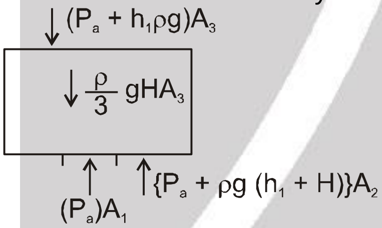
$\Rightarrow V = \frac{165}{1100} = 0.15\ m^3$
Reading on spring balance under water is :
$W_{app} = [165 \times 10] - 0.15\,[10^3]\,[10] = 150$ N

10. Just after the string is cut :
$a = \frac{150}{165} = 0.91\ m/s^2$ **Ans.**

EXERCISE-3

PART - I

1. $A_1 = \pi r^2$ = area of base of cylinder in air
$A_2 = 3\pi r^2$ = area of base of cylinder in water
$A_3 = 4\pi r^2$ = cross-section area of cylinder



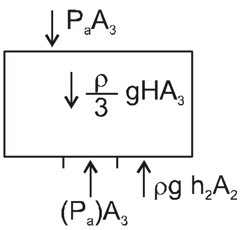
from the equilibrium of block (see diagram)
Equating the forces, we get

$(P_a + \rho g h_1)A_3 + \frac{\rho}{3} gH\ A_3 = (P_a)A_1 + [P_a + \rho g(h_1 + H)]A_2$

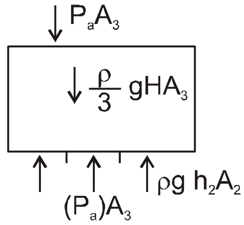
On solving

$h_1 = \frac{5}{3}H$

2. (A) $P_aA_3 + \frac{\rho}{3}A_3Hg = P_aA_3 + \rho g h_2 A_2$



from the equilibrium of block (see diagram)

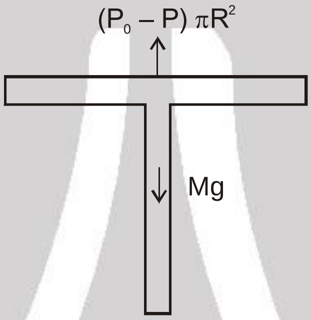


$h_2 = 4H/9$.

3. For $h_2 < 4h/9$ cylinder does not move up because further bouyant force decreases while the weight of block remains same.

4. Since it is open from the top, the pressure will be P_0

5. Resultant force on the piston is zero (Let pressure in air be P)
From the equilibrium of the piston



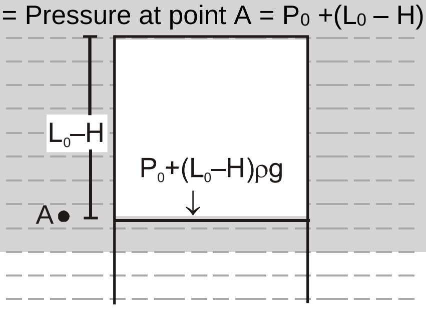
$(P_0 - P)\ \pi R^2 = Mg$

$P = P_0 - \dfrac{Mg}{\pi R^2}$

From the conservation of moles of air : $P_1V_1 = P_2V_2$, it follows that
$P_0 \ .\ 2L = Px$

$\therefore\ x = \dfrac{P_0.2L}{P} = \dfrac{P_0 2L}{P_0 - \dfrac{Mg}{\pi R^2}}$

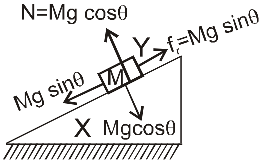
6. Pressure in air inside cylinder = Pressure at point A = $P_0 + (L_0 - H)\ \rho g$



PV = constant in the air inside the cylinder
$\therefore\ P_0L_0 = [P_0 + (L_0 - H)\rho g]\ (L_0 - H)$
$P_0\ (L_0 - H) + \rho g\ (L_0 - H)^2 - P_0L_0 = 0$

7. As the stream falls down, its speed will increase and cross-section area will decrease.
Thus it will become narrow.
Similarly as the stream will go up, speed will decrease and cross-section area will increase.
Thus it will become broader.
Hence statement-1 is correct and statement-2 is correct explanation also.
Ans. (A)

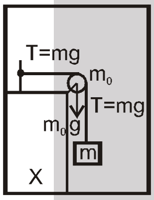
8. (p)



Net force on Y due to X = $\sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2}$ = mg

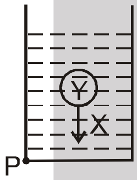
(B) As the inclined is fixed. So, gravitational P.E. of X is constant
(C) As K.E. is constant and P.E. of Y is decreasing. So mechanical energy of (X + Y) is decreasing.
(q)
(A) force on Y due to X will be greater than mg which is equal to (Mg + repulsion force)
(B) As the system is moving up, P.E. of X is increasing.
(C) Mechanical energy of (X + Y) is increasing
(D) Torque of the weight of Y about point P = 0

(r)



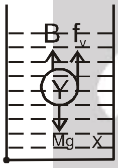
(A) force on Y due to X = $\sqrt{[(m + m_0)g]^2 + (mg)^2}$
(B) As the system moves down, gravitational P.E. of X decreases
(C) As the system moves down, total mechanical energy of (X + Y) also decreases
(D) $\tau_P \neq 0$

(s)



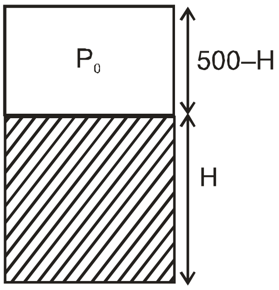
(A) force on Y due to X = Buoyancy force which is less than mg
(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X increases.
(C) As there is no non–conservative force, so total mechanical energy of X + Y remains conserved.
(D) $\tau_p \neq 0$

(t)



(A) As the sphere is moving with constant velocity
B + f_v = Mg
so force on Y due to X is B + f_v = mg
(B) As the sphere moves down, that volume of water comes up, so gravitational P.E. of X will increase
(C) Increase in mechanical energy
w_{fr} = –ve
(D) $\tau_p \neq 0$

9.



P_0 = atmospheric pressure

$P + 200 \times 10^{-3} \times 1000 \times 10 = P_0$ ....(1)

$P_0 (500 - H) = P \,.\, (300 \text{ mm})$

$\Rightarrow \quad P = \dfrac{P_0(500 - H) \text{ mm}}{300 \text{ mm}}$ ....(2)

from (1) and (2)

$\dfrac{P_0(500 - H)}{300} + 2000 = P_0$

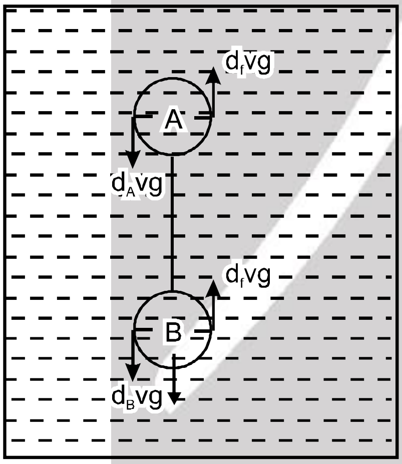
$\dfrac{10^5(500 - H)}{300} + 2000 = 10^5$

$\Rightarrow \quad 5 \times 10^7 - H \times 10^5 + 6 = 300$

$\Rightarrow \quad H = 206 \text{ mm}$

fall in height = 6 mm **Ans. 6**

10.



For equilibrium

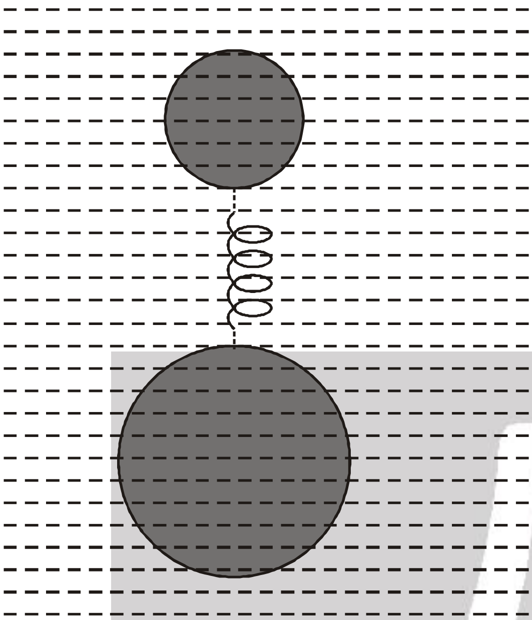
$d_A vg + d_B vg = d_F vg + d_F vg$

$\Rightarrow \; d_F = \dfrac{d_A + d_B}{2} \quad \Rightarrow$ Option (D) is correct

to keep the string tight

$d_B > d_F$ and $d_A < d_F$

11.



On small sphere

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \quad ....(i)$$

on second sphere (large)

$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \quad ....(ii)$$

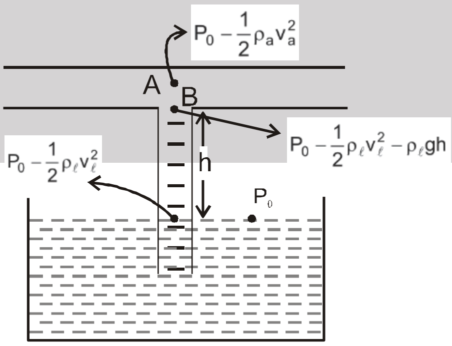
by equation (i) and (ii)

$$x = \frac{4\pi R^3 \rho g}{3k}$$

12. $A_1 V_1 = A_2 V_2$ $\qquad$ $A_1 = 400\ A_2$

$400\ (5 \times 10^{-3}) = V_2 \quad \Rightarrow \quad \boxed{V_2 = 2 \quad m/s}$ $\qquad$ (C)

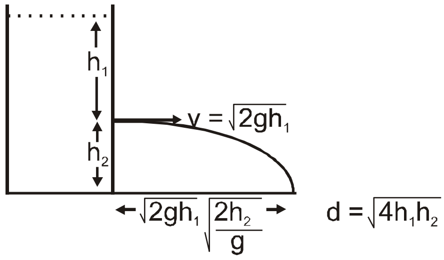
13. Pressure at A and B will be same



$$P_0 - \frac{1}{2}\rho_a v_a^2 = P_0 - \frac{1}{2}\rho_\ell v_\ell^2 - \rho_\ell gh$$

$$v_\ell = \sqrt{\frac{\rho_a}{\rho_\ell} v_a - 2gh}$$

14. Match the column
When lift is at rest :



(P) $g_{eff} > g$ $\quad$ $d = \sqrt{4h_1h_2} = 1.2$ m

(Q) $g_{eff} < g$ $\quad$ $d = \sqrt{4h_1h_2} = 1.2$ m

(R) $g_{eff} = g$ $\quad$ $d = \sqrt{4h_1h_2} = 1.2$ m

(S) $g_{eff} = o$ $\quad$ No water leaks out of the jar.

PART - II

1. Since solid ball floats in between the two liquids hence $\rho_1 < \rho_3 < \rho_2$

2. For equilibrium, weight should be balanced by buoyant force.
density of oil < density of water
and ball should be in between oil and water.

3. Diameter = 8×10^{-3} m
v = 0.4 m/s

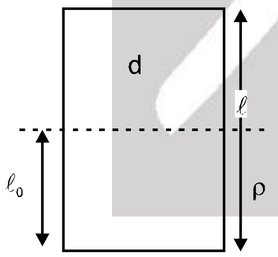
$v = \sqrt{u^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2} = 2$ m/s

$A_1v_1 = A_2v_2$

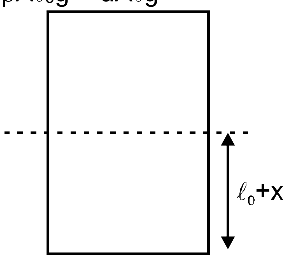
$$\pi\left(\frac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \frac{d^2}{4} \times 2$$

$d \approx 3.6 \times 10^{-3}$ m.

4.



At equilibrium
$F_b = mg$
$\rho A\ell_0 g = dA\ell g$ ....(i)



Restoring force,
$F = mg - F_b'$

$F = mg - \rho A(\ell_0 + x)g$

$dA\ell a = dA\ell g - \rho A\ell_0 g - \rho gAx$

$a = -\dfrac{\rho g}{d\ell}x$

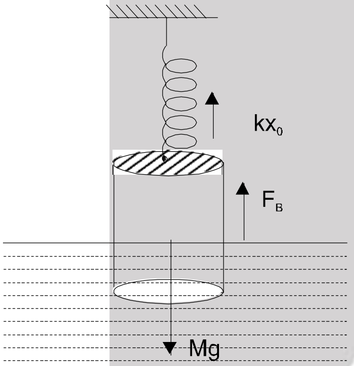
$\omega = \sqrt{\dfrac{\rho g}{d\ell}}$

$T = 2\pi\sqrt{\dfrac{\ell d}{\rho g}}$ ....(i)

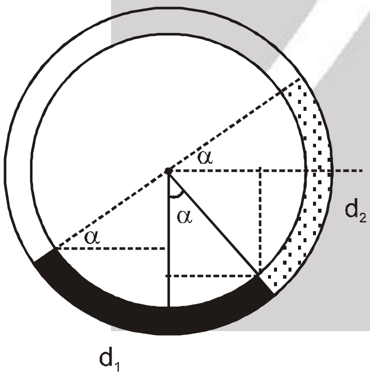
5. $kx_0 + F_B = mg$

$kx_0 + \dfrac{L}{2}\sigma Ag = Mg$

$x_0 = \dfrac{Mg - \dfrac{\sigma LAg}{2}}{k} = \dfrac{Mg}{k}\left(1 - \dfrac{\sigma LA}{2M}\right)$



6.



$R\sin\alpha\ d_2 + R\cos\alpha\ d_2 + R(1 - \cos\alpha)d_1$

$= R(1 - \sin\alpha)\ d_1$

$(\sin\alpha + \cos\alpha)\ d_2 = d_1(\cos\alpha - \sin\alpha)$

$\Rightarrow \dfrac{d_1}{d_2} = \dfrac{1 + \tan\alpha}{1 - \tan\alpha}$

HIGH LEVEL PROBLEMS (HLP)

1. In elastic collision with the surface, direction of velocity is reversed but its magnitude remains the same. Therefore, time of fall = time of rise.
or time of fall = $t_{1/2}$
Hence, velocity of the ball just before it collides with liquid is

$$v = g\frac{t_1}{2} \quad ....(1)$$

Retardation inside the liquid

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$= \frac{Vd_L g - Vdg}{Vd} = \left(\frac{d_L g - d}{d}\right) g (V = \text{volume of ball}) \quad ....(2)$$

Time taken to come to rest under this retardation will be

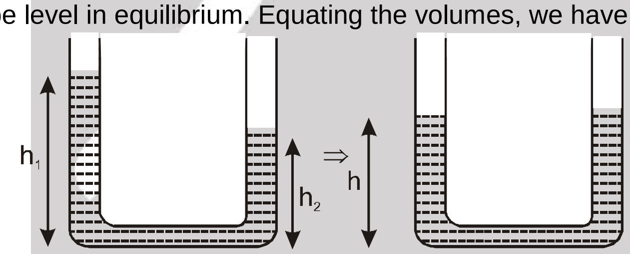
$$t = \frac{v}{a} = \frac{gt_1}{2a} = \frac{gt_1}{2\left(\frac{d_L - d}{d}\right) g} = \frac{dt_1}{2(d_L - d)}$$

Same will be the time to come back on the liquid surface.
Therefore,
(a) t_2 = time the ball takes to came back to the position from where it was released

$$= t_1 + 2t = t_1 + \frac{dt_1}{d_L - d} = t_1\left[1 + \frac{d}{d_L - d}\right] \quad \text{or} \quad t_2 = \frac{t_1 d_L}{d_L - d}$$

(b) The motion of the ball is periodic but not simple harmonic because the acceleration of the ball is g in air and $\left(\frac{d_L - d}{d}\right)$ g inside the liquid which is not proportional to the displacement, which is necessary and sufficient condition for SHM.
(c) When $d_L = d$, retardation or acceleration inside the liquid becomes zero (upthrust = weight). Therefore, the ball will continue to move with constant velocity $v = gt_1 / 2$ inside the liquid.

2. Let h be level in equilibrium. Equating the volumes, we have



$Ah_1 + Ah_2 = 2Ah$

$\therefore \quad h = \left(\frac{h_1 + h_2}{2}\right)$

Work done by gravity = $U_i - U_f$

$$W = \left(m_1 g\frac{h_1}{2} + m_2 g\frac{h_2}{2}\right) - (m_1 + m_2)g\frac{h}{2}$$

$$= \frac{Ah_1\rho gh_1}{2} + \frac{Ah_2\rho gh_2}{2} - [Ah_1\rho + Ah_2\rho]g\left(\frac{h_1 + h_2}{4}\right)$$

Simplifying this, we get

$$W = \frac{\rho Ag}{4}(h_1 - h_2)^2$$

3.

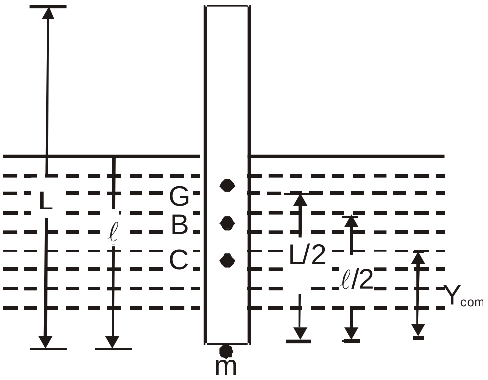


Figure (1)

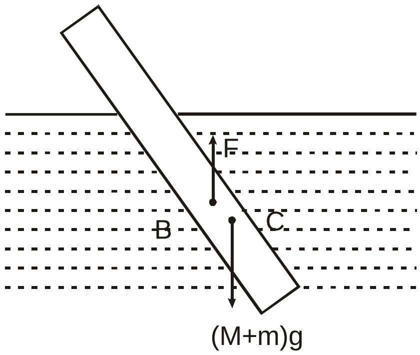


Figure (2)

Let M = Mass of stick = $\pi R^2 \rho L$
ℓ = Immersed length of the rod
G = COM of rod
B = Centre of buoyant force (F)
C = COM of rod + mass (m)
Y_{com} = Distance of C from bottom of the rod

Mass m should be attached to the lower end because otherwise B will be below G and C will be above G and the torque of the couple of two equal and opposite forces F and (M + m) g will be counter clockwise on displacing the rotational equilibrium. See the figure 3 given alongside.

For vertical equilibrium

Mg + mg = F (upthrust)

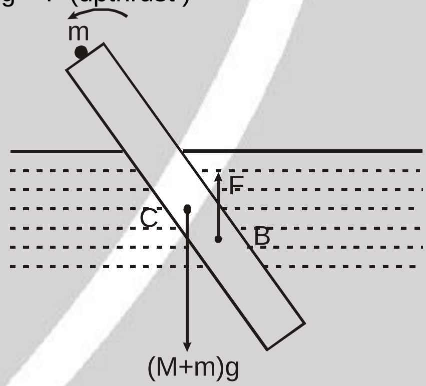


Figure (3)

or $(\pi R^2 Lg\rho) + mg = (\pi R^2 \ell)\sigma g$

$\therefore \quad \ell = \left(\dfrac{\pi R^2 L\rho + m}{\pi R^2 \sigma}\right)$ .....(1)

Position of COM (of rod + m) from bottom

$Y_{com} = \dfrac{M\frac{L}{2}}{M+m} = \dfrac{(\pi R^2 L\rho)\frac{L}{2}}{(\pi R^2 L\rho)+m}$ …..(2)

Centre of buoyancy (B) is at a height of $\frac{\ell}{2}$ from the bottom.

We can see from figure (2) that for rotational equilibrium of the rod, B should either lie above C or at the same level of B.

Therefore $\frac{\ell}{2} \geq Y_{COM}$ or $\dfrac{\pi R^2 L\rho + m}{2\pi R^2 \sigma} \geq \dfrac{(\pi R^2 L\rho)\frac{L}{2}}{(\pi R^2 L\rho)+m}$

or $m + \pi R^2 L\rho \geq \pi R^2 L\sqrt{\rho\sigma}$ or $m \geq \pi R^2 L(\sqrt{\rho\sigma} - \rho)$

$\therefore$ Minimum value of m is $\pi R^2 L(\sqrt{\rho\sigma} - \rho)$ **Ans.**

4. (a) (i) Considering vertical equilibrium of cylinder :
Weight of cylinder = upthrust due to upper liquid
+ upthrust due to lower liquid

Note that h_1 and $h_2 \neq \frac{H}{2}$

$$\therefore \left(\frac{A}{5}\right)(L)\, D.g = \left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)(d)\, g + \left(\frac{A}{5}\right)\left(\frac{L}{4}\right)(2d)\,(g)$$

$$\therefore \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$D = \frac{5}{4}d$$

(ii) Considering vertical equilibrium of two liquids and the cylinder.
$(P - P_0)\, A$ = weight of two liquids + weight of cylinder

$$\therefore \quad P = P_0 + \frac{\text{weight of two liquids} + \text{weight of cylinder}}{A} \qquad ...(1)$$

Now , weight of cylinder

$$= \left(\frac{A}{5}\right)(L)\,(D)\,(g) = \left(\frac{A}{5}Lg\right)\left(\frac{A}{5}d\right)$$

$$= \frac{ALdg}{4}$$

Weight of upper liquid $= \left(\frac{H}{2}Adg\right)$ and

Weight of lower liquid $= \frac{H}{2}\, A\,(2d)g = HAgd$

$\therefore$ Total weight of two liquids $= \frac{3}{2}HAdg$

$\therefore$ From Eq. (1) pressure at the bottom of the container will be

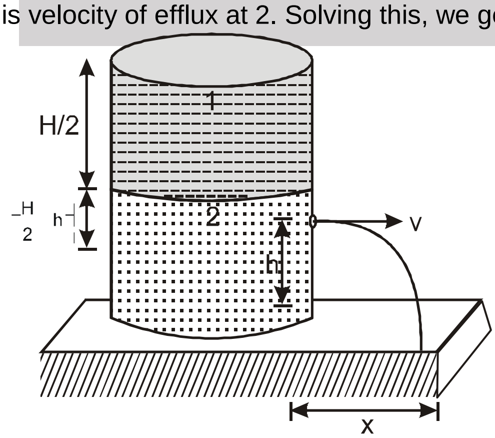
$$P = P_0 + \frac{\left(\frac{3}{2}\right)HAdg + \frac{ALdg}{4}}{A}$$

or $$P = P_0 + \frac{dg(6H + L)}{4}$$

(b) (i) Applying Bernoulli`s theorem at 1 and 2

$$P_0 + dg\left(\frac{H}{2}\right) + 2dg\left(\frac{H}{2} - h\right) = P_0 + \frac{1}{2}(2d)\, v^2$$

Here, v is velocity of efflux at 2. Solving this, we get



$$v = \sqrt{(3H - 4h)\frac{g}{2}}$$

(ii) Time taken to reach the liquid to the bottom will be

$$t = \sqrt{2h/g}$$

\ Horizontal distance x travelled by the liquid is

$$x = vt = \sqrt{\left(3H - 4h\right)\frac{g}{2}}\left(\sqrt{\frac{2h}{g}}\right)$$

$$x = \sqrt{h(3H - 4h)}$$

(iii) For x to be maximum

$$\frac{dx}{dh} = 0$$

or $$\frac{1}{2\sqrt{h(3H-4h)}}\,(3H - 8h) = 0$$

or $$h = \frac{3H}{8}$$

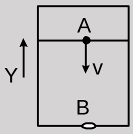
Therefore, x will be maximum at $h = \frac{3H}{8}$.

The maximum value of x will be

$$x_m = \sqrt{\left(\frac{3H}{8}\right)3H - 4\left(\frac{3H}{8}\right)}$$

$$x_m = \frac{3}{4}H$$

5. Let the velocity of efflux of mercury coming out of hole be ν at an instant mercury level in container is y. At same instant the speed of top surface of fluids v. from equation of continuity



$$\frac{S}{n}\,v = Sv \qquad ....(1)$$

$\because \quad \frac{S}{n} \ll S \qquad \therefore \qquad \nu \gg v$

applying Bernoulli's theorem between A and B

$$Yeg + \frac{1}{2}ev^2 = P_{atm} + \frac{1}{2}e\nu^2$$

$\because$ $v \ll \nu$ higher powers of v can be neglected and $P_{atm} = h_0eg$

$\therefore$ $$\nu = \sqrt{2g(Y - h_0)} \qquad ....(2)$$

Hence mercury flows out of both till $y = h_0$.
from equation (1)

$$v = -\frac{dy}{dt} = \frac{1}{h}\nu = \frac{1}{h}\sqrt{2g(Y - h_0)}$$

or $$= \frac{-dy}{\sqrt{2gY - h_0}}\;\frac{1}{h}\,dt$$

integrating between limits

at $l = 0$ $\quad y = n \quad$ and $t = T$, $Y = h_0$; $-\int_n^{h_0}\frac{dy}{\sqrt{2g\;(Y - h_0)}} = \frac{1}{n}\int_0^T dt$; $t = n\sqrt{\frac{2}{g}(h - h_0)}$

6. Bernoulli's equation between A and B gives

$$\frac{p_A}{\rho_a} = \frac{p_B}{\rho_a} + \frac{v^2}{2} \Rightarrow \qquad v^2 = 2\left[\frac{p_A - p_B}{\rho_a}\right]$$

Also equating pressures at horizontal level of E

$p_A + \rho_a gy + \rho_a gh = p_B + \rho_a gy' + \rho_a gy + \rho_m gh.$

$\Rightarrow$ $p_A + \rho_a gh = p_B + \rho_m gh$ $[\because y'\ 0]$

$p_A - p_B = (\rho_m - \rho_a)\ gh.$

$$v^2 = \frac{2(\rho_m - \rho_a)gh}{\rho_a}$$

$$\mathbf{v} = \sqrt{\frac{2(\rho_m - \rho_a)gh}{\rho_a}}$$ **Ans.**

7. Weight of sphere + chain = $(m + \lambda h)g$

Buoyant force = $(3m + \frac{\lambda h}{7})\ g$

for equilibrium, weight = Buoyant force or , $m + \lambda h = 3m + \frac{\lambda h}{7}$ or $h = \frac{7m}{3\lambda}$

8. Apply Bernolli's equation b/w point P & D

$$P_{atm} + \rho g h_1 + \frac{1}{2}\rho V_p^2 = P_{atm} + \rho g(o) + \frac{1}{2}\rho v_D^2$$

[Assume zero level D]

$V_p \approx 0$ Area of cross section is very large

$V_D = \sqrt{2gh_1}$

Since area at C is half than area at D
so according to continuity equation

$V_C A_C = V_D A_D \Rightarrow V_C = 2V_D = 2\sqrt{2gh_1}$

Now for point P & C according to Bernolli's equation.

$$P_p + \rho g h_1 + \frac{1}{2}\rho V_p^2 = P_C + \rho g(0) + \frac{1}{2}\rho . V_C^2$$

$V_p \approx 0$

$\Rightarrow$ $P_{atm} + \rho g h_1 = P_C + \frac{1}{2}\rho.(2\sqrt{2gh_1})^2$

$\Rightarrow$ $P_{atm} = P_C + 3\rho g h_1$ ....(i)

for point E

$P_E = P_C + \rho g h_2 = P_{atm}$ ....(ii)

from (i) & (ii)

$3\rho g h_1 = \rho g h_2$

$\Rightarrow$ $\mathbf{h_2 = 3h_1}$ **Ans.**